OPTIMIZATION OF MACHINING CONDITIONS IN MULTIPASS TURNING WITH CONSTRAINTS- A SIMPLIFIED APPROACH

by

RAJIVE GUPTA

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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RAJIVE GUPTA

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CERTIFICATE

It is certified that the work contained in the thesis entitled "**Optimization of Machining Conditions in Multipass Turning with Constraints - A Simplified Approach**" by Rajive Gupta, has been carried out under our supervision, and that this work has not been submitted elsewhere for a degree.

Prof. Kripa Shanker
Department of Industrial and
Management Engineering
Indian Institute of Technology, Kanpur

Prof. G.K. Lal
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

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Dedicated to my parents

Synopsis

The machining conditions for a particular operation are still to a large extent obtained from manufacturer's data or machining handbooks which provide rather conservative estimates. These machining conditions are normally obtained on the basis of economic criteria such as minimum production cost or time per piece, or maximum profit rate. More realistic conditions can be obtained by considering economic optimum under technical constraints of real life machining such as maximum allowable cutting power, cutting force, surface roughness etc. Machining conditions thus obtained will be the optimal conditions.

Because of non-availability of nonlinear programming (NLP) algorithms, earlier efforts for finding optimal machining conditions have been limited to differential calculus based methods or numerical methods. The number of constraints considered were only a few because inclusion of large number of constraints involve development of complex logic for searching the optimum. Later NLP methods, both unconstrained and constrained, have been used for evaluating the optimal machining conditions. Geometric programming (GP) has also been extensively used for obtaining optimal machining condition since it was observed that a large number of technical constraints could be represented by monomials or single term polynomials.

The real fruits of optimization of machining conditions have not been reaped because of two difficulties. (1) The unknown number of passes at multipass problem formulation stage and (2) the requirement of a feasible or non-feasible starting solution during optimization stage which again requires solution of another optimization problem. The first problem has been solved to some extent by taking a small depth of cut in finish pass and trying all possible number of even passes. Another solution is to solve the multipass turning problem using dynamic programming (DP) approach where each pass is considered a stage. In this case the total depth is divided into a number of segments and single pass problems are recursively solved to reach the optimal solution. The optimal solution contains the optimal number of passes alongwith the number of segments falling into each pass, and cutting speeds and feeds. Any effort to reduce the segment size squarely increases the number of single pass solutions. This method, therefore, requires more computation time and is not

popular. For the solution of second problem sequential quadratic programming method (SQP) may be used. This method can be started from any arbitrary point but it is not always easy to implement.

The present work has been carried out mainly for removing the above mentioned difficulties. The effort has been to find the optimal number of passes uniquely. The minimum production cost per piece criterion has been adopted for the determination of optimal machining conditions. The multipass problem has not been solved by taking all passes together, instead the solution has been synthesized using single pass solutions. While attempting to get the solution the following points have also been considered:

- Selection of computationally efficient and easy to implement methodology for the solution of single pass turning problem. The methodology selected should not necessarily require a feasible or a non-feasible starting point and should not depend upon the optimization parameters.

- Parametric analysis of single pass methodology so that solution to new problems generated because of changed workpiece dimensions can be easily computed. This will help in reduction of computation time during application of DP approach to multipass turning problem.

- A method for logical selection of depth of cut for finish pass and optimal distribution of depth of cut during finish as well as rough passes.

- The concept of uneven passes as much of the research work is still concentrated towards even passes.

- Development of a model for optimal clubbing of machinable volumes, horizontally, vertically or in circular order for achieving minimum production cost.

A detailed review of existing literature is presented in Chapter 1. The topics covered include CAPP systems for rotational parts, various tool life equations, different optimization criteria, optimization techniques, etc. The papers have also been summarized in a tabular form which indicates important factors such as single or multipass turning, economic criterion, deterministic or probabilistic model, unconstrained or constrained turning constraints used in the model, optimization approach, solution methodology, decision variables and type of tool life equation used for constructing the turning model.

The various time components viz. preparation time, idling time, machining time, tool changing time and the corresponding costs have been defined in Chapter 2. The technical constraints such as cutting force, cutting power, surface finish, etc. have also been discussed

in this chapter. It has been observed that majority of constraints are monomials. Therefore, a general multipass cost minimization problem has been formulated using extended Taylor's tool life equation and monomial constraints. Assuming that one finish and a few even rough passes are required to remove total material, the minimum and maximum total number of passes are decided on the basis of allowable ranges of depths of cut for finish as well as rough passes. The multipass problem is repeatedly solved for all possible total numbers of passes and the number giving the least minimum production cost or time per piece is declared optimal. The solution process is further simplified by providing depth of cut distribution also along with number of passes as input to the multipass problem. The common practice is to use a small depth of cut for finish pass and remove the remaining depth in even member of rough passes. The multipass problem is now decomposed into a number of single pass problems which are easier to solve. At this stage assumptions of equal workpiece diameter and/or equal tool life in all passes are also used to reduce the number of single pass problems. The general multipass turning problems may also be solved using the dynamic programming (DP) approach which gives uniquely optimal number of passes using single pass solutions. The total depth of cut is divided into a number of segments and the optimal solution contains optimal number of passes, the number of segments falling into each pass, and the cutting speed as well as feed for each pass.

Out of two approaches, optimizing all the passes simultaneously or using single pass solutions to synthesize results for multipass turning problem, the later one has been adopted in the present work as it is easier to obtain single pass solutions. The common practice is to search the optima in speed-feed plane. Various NLP methods use different techniques to formulate and update search vector comprising of speed and feed components and then use single dimensional search methods to obtain optima along these search vectors. It has been found that at a given tool life single pass production cost for a given depth of cut is minimized by using maximum allowable feed under constraints at that particular tool life. Therefore, instead of finding a search vector and updating it in speed-feed plane, a one dimensional search along tool life may be performed to find out the optimal tool life and feed corresponding to minimum production cost. The cutting speed is obtained using extended Taylor's tool life equation. Thus, the process of searching for optima under the constraints is immensely simplified.

The single pass solution methodology has been described in Chapter 3. Parametric analysis for this methodology has also been discussed. This analysis helps in finding the solution for new single pass turning problems with changed workpiece dimensions for the same depth of cut without solving the new problem.

In this chapter two solution models have been presented. The first one uses a rather new concept of assumed maximum number of passes and depth of cut verses minimum production cost trends. The minimum production costs for a series of feasible depths of cut are obtained for finish as well as rough passes. One finish pass and a large number of rough passes are assumed to remove the total depth of cut. A binary variable is associated with each feasible depth in series in finish as well as rough passes. A linear integer programming model is formed using the feasible depths in series, the minimum production costs corresponding to the depths, and binary variable for the assumed maximum number of passes. The problem is solved using LINDO optimizer and optimal number of passes are obtained by counting non-zero binary variables in the solution.

The second model is based on dynamic programming approach and is already available in the literature. It could not be explored fully because any effort to reduce the segment size squarely increases the number of single pass solutions. The parametric analysis for single-pass solution methodology suggested in this chapter reduces the computational work and thus helps in reduction of segment size to 0.1 mm. Earlier workers had taken the segment size as 1 mm.

The numerical results for a few straight multipass turning examples have been presented in Chapter 4. It has been observed during the use of integer programming model that uneven passes yield lower minimum production cost. DP approach gives optimal solution with better accuracy and is capable of handling the general multipass problem without making assumptions such as equal workpiece diameter or equal tool life in all passes. DP approach along with single pass solution methodology has been compared with Sequential Unconstrained Minimization Technique (SUMT) with Davidon Fletcher Powell's method (DFP), Generalized Reduced Gradient (GRG) method and Sequential Quadratic Programming (SQP) method. It has been observed that for a general multipass turning example, DP approach is easier to implement than any other method and it does not require a feasible or nonfeasible starting point. Further, this approach requires less computer time than SUMT with DFP and GRG and works equally well as SQP.

Rough turning of stepped shaft involves decision on horizontal, vertical, or circular clubbing of machinable volumes. A model has been suggested to perform logical clubbing of volumes in Chapter 5. This model has been solved using LINDO optimizer. The results obtained for two shaft which are similar in shape but of different dimensions show that different optimal grouping may be obtained even if the shape of shafts appear to be similar. Further, the optimal clubbing of volumes obtained using mathematical model yields better solution than that obtained using horizontal clubbing, or using a heuristic of ascending

minimum production cost/unit volume.

The conclusions and scope for future work have been presented in Chapter 6. There are two basic issues involved in the solution of multipass turning problems. The first is to obtain optimal number of passes and second is to treat all the passes simultaneously, or decompose the multipass problem into single pass problems. It is easier to solve single pass problems. Therefore, second issue has been taken care by suggesting a simple single pass solution methodology. The parametric analysis for this methodology has been performed for the first time. A new integer programming model has been suggested for obtaining the optimal number of passes as well as the optimal depth of cut distribution. The available DP model has been adopted because it uniquely gives optimal number of passes. During application of dynamic programming approach to multipass turning operation the current practice is to optimize the single pass turning problems each time they are formed due to varying workpiece diameter even if the depth of cut remainsthe same. Once solution to a single pass turning problem for a given depth of cut is available the parametric analysis helps in getting the solution of these newly formed problems of same depth of cut with varying workpiece diameter in each pass through simple calculations. Both the integer programming model and DP approach have been useful in logically deciding the depth of cut for finish pass which used to be kept at some small or least value during the finish pass. Further, these models have helped in obtaining better minimum production cost by exploring uneven passes. Parametric analysis for single pass solution method has been very useful in reducing the segment size of depth of cut and computational effort during DP approach.

The present work considers monomial constraints and may be extended to explore the treatment of nonmonomial constraints. The turning operation has been taken for simplicity. It will be interesting to know if other machining operations such as milling, drilling, reaming, boring, etc. can also be treated using the same methodology.

Contents

List of Symbols

a	binary variable used for grouping,of machinable volumes
A	matrix of binary variables a_{eg}
$A0_1$, $A0_2$	constants in production cost function represented terms of tool life, feed and depth of cut
$A0'_1$, $A0''_1$	constants $A0_1$ defined per unit diameter, and per unit diameter and length respectively
$cost_{kq}$	minimum production cost for machining for k division when q^{th} depth of cut is machined in rough pass
$cost_{min\ k}$	minimum of all $cost_{kq}$ for a given k
C_a	tool approach and retreact time per piece, \$/piece
C_c	tool change cost per piece, \$/piece
C_m	machining cost per piece, \$/piece
C_p	cost of preparation per piece, \$/ piece
C_t	tool cost per piece, \$/piece
C_{to}	total cost per piece, \$/piece
C_{tovar}	variable minimum production cost per piece, \$/piece
C'_{tovar}, C''_{tovar}	C_{tovar} defined as per unit diameter, and per unit diameter and length
C_T	tool life constant in extended Taylor's tool life equation
C_{T0}	tool life constant in Taylor's tool life equation
C_{T1}	tool life constant in Colding's tool life equation
d	depth of cut in a pass, mm
d_a	diametral accuracy, mm
d_r	depth of cut in rough pass, mm
d_s	depth of cut in rough pass, mm
d_{ti}	remaining depth of cut before a pass i, mm
d_{cz}	depth of zone with compressive stress, mm
d_{pz}	depth of plastcally deformed zone, mm
D_0	original outside diameter of workpiece, mm
D_f	final outside diameter of workpiece, mm

D_h	internal diameter of hollow workpiece, mm
f	feed, mm/rev
f_{minlim}	limit of minimum feed in satisfactory chip-breaker region, mm/rev
f_{maxlim}	limit of maximum feed in satisfactory chip-breaker region, mm/rev
F_c	cutting force, KN
F_r	radial force, KN
F_{th}	tool thrust, KN
G	function used to define equivalent chip thickness
h	peak-to-valley height of surface finish, mm
h_1, h_2	constants related to tool approach and retreact time
h_e	equivalent chip thickness
k_l	direct labour and overhead cost, \$/min
k_m	electricity and cutting oil charges for a machine, \$/min
k_{mc}	raw material cost per piece, \$/piece
k_t	cost per tool edge, \$/edge
key_k	key for backtracking depth of cut distribution in DP model
$K0_1$, $K0_2$	constants associated with machining cost and tool cost respectively in production cost function
K_0	constant in tool life equation
$K_1, K_1^{'}$	constant in cutting force constraint and equation
$K_2, K_2^{'}$	constants in cutting power constraint and equation
$K_3, K_3^{'}$	constants in spindle torque constraint and equation
K_4	constant in tool life limitation constraint
$K_5, K_5^{'}$	constant in tool thrust constraint and equation
K_6, $K_6^{'}$	constant in tensile stress on rake face constraint and equation
$K_{71}^{'}$, $K_{71}^{''}$, $K_{71}^{'''}$	constants in adhesive tool wear rate equation
$K_{72}^{'}$, $K_{72}^{''}$, $K_{72}^{'''}$	constants in diffusion tool wear rate equation
K_8, $K_8^{'}$	constants in primary shear zone temperature constraint and equation
$K_9, K_9^{'}$	constants in chip-tool interface temperature constraint and equation
K_{10}	constants in stable cutting region contraint
K_{12}, $K_{12}^{'}$	constants in surface finish constraint and equation
K_{13}, $K_{13}^{'}$	constants in depth of zone with residual compreessive stress constraint and equation
K_{14}, $K_{14}{'}$	constants in plastically deformed depth constraint and equation
$K_{15}, K_{15}^{'}$	constants in residual tensile stress constraint and equation
$K_{16}, K_{16}^{'}$	constants in residual compressive stress constraint and equation
$K_{17}, K_{17}^{'}$	constants in diametral accuracy constraint and equation
l_c	chip engagement length, mm

L	length of workpiece, mm
L_o	overhang length of workpiece, mm
L_w	allowable wear-land due to adhesion
L_{w1}	wear rate due to adhesion
L_{w2}	wear rate due to diffusion
M	spindle tarque, Nm
n	number of rough passes
n_p	total number of passes
n_c	number of monomial constraints in a rough or finish pass
n_{pc}	number of pieces
n_t	number of tool changes per part
n_1,n_2,n_3	exponents of tool life, feed and depth of cut in extended Taylor's tool life equation
n_4,n_5	exponents of tool life and equivalent chip thickness in Colding's tool life equation
n_6,n_7,n_8	constants used in approximation of G function.
N_c	number of cuts of equal depth in one tool life
P	cutting power, kW
P_r	profit rate, $/min
r	nose radius of tool, mm
R_a	surface roughness CLA value, μm
S	selling price per piece, $/piece
t_a	tool approach and retreact time per piece, min/piece
t_c	tool changing time per edge, min/edge
t_e	tool changing time per piece, min/piece
t_m	machining time per piece, min/piece
t_p	preparation time per piece, min/piece
t_{to}	total production time per piece, min/piece
T	tool life, min
T_{mc}	tool life corresponding to unconstrained minimum production cost, min
T_{mt}	tool life corresponding to unconstrained minimum production time, min
T_{pr}	tool life corresponding to unconstrained maximum profit rate, min
T_r	reference tool life, min
v	speed, m/min
v_{mc}	speed corresponding to T_{mc}, m/min
v_{mt}	speed corresponding to T_{mt}, m/min
v_{pr}	speed corresponding to T_{pr}, m/min
v_r	reference speed, m/min

V	candidate volume of a stepped shaft
x	binary variable associated with machinable volume of a stepped shaft
X	binary variable used in integer programming model
$\alpha_0, \beta_0, \gamma_0$	exponent of speed, feed and depth of cut in extended tool life constraint or equation
$\alpha_1, \beta_1, \gamma_1$	exponent of speed, feed and depth of cut in cutting force constraint or equation
$\alpha_2, \beta_2, \gamma_2$	exponent of speed, feed and depth of cut in cutting power constraint or equation
$\alpha_3, \beta_3, \gamma_3$	exponent of speed, feed and depth of cut in spindle torque constraint or equation
β_5, γ_5	exponent of feed and depth of cut in tool thrust constraint or equation
α_6, β_6	exponent of speed and feed in tensile stress on rake face constraint or equation
$\alpha_{71}, \nu_{71}, \nu_{72}, \delta_{71}$	constants used in adhesion wear and diffusion wear equations
$\alpha_8, \beta_8, \gamma_8$	exponent of speed, feed and depth of cut in primary shear zone temperature constraint or equation
$\alpha_9, \beta_9, \gamma_9$	exponent of speed, feed and depth of cut in chip tool interface temperature constraint or equation
$\alpha_{10}, \beta_{10},$	exponent of speed and feed in stable cutting region constraint or equation
$\alpha_{12}, \beta_{12}, \gamma_{12}$	exponent of speed, feed and depth of cut in surface finish constraint or equation
$\alpha_{13}, \beta_{13}, \gamma_{13}, \delta_{13}$	exponent of speed, feed, depth of cut and nose radius in depth of zone with compressive stress constraint or equation
$\alpha_{14}, \beta_{14}, \gamma_{14}, \delta_{14}$	exponent of speed, feed, depth of cut and nose radius in chip-tool interface temperature constraint or equation
$\alpha_{15}, \beta_{15}, \gamma_{15}, \delta_{15}$	exponent of speed, feed, depth of cut and nose radius in residual tensile stress constraint or equation
$\alpha_{16}, \beta_{16}, \gamma_{16}, \delta_{16}$	exponent of speed, feed, depth of cut and nose radius in residual compressive stress constraint or equation
σ_c	residual compressive stress, N/sq. mm
σ_t	tensile stress on rake face, N/sq. mm
σ_{tr}	residual tensile stress, N/sq. mm
θ	temperature in primary shear zone, °C
θ_s	temperature in chip-tool interface, °C
$\theta_{softening}$	softening temperature of tool, °C

ϕ_1, ϕ_2	angles defining satisfactory chip-breaker region
ψ	side cutting edge angle
δ	chip slenderness ratio
Δ	deflection of workpiece, mm
ζ	ratio of tool life for rough pass to finish pass

Subscripts

i	i^{th} pass
e	e^{th} machinable volume
g	g^{th} candidate volume
j	j^{th} constraint
q	q^{th} division of depth of cut
min	minimum
max	maximum

Superscripts

opt	corresponding to optimal value for total passes
optr	corresponding to optimal value of continuous number of passes
$*$	corresponding to optimal value for a single rough or finsh pass

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Chapter 1

Introduction and Literature Review

1.1 INTRODUCTION

The world of machining has been changing continuously and rapidly. Most of the earlier changes were brought about by advancements in machining technology and development of new tool materials. Rapid changes in the past few decades can be attributed mainly to automation leading to the development of programmable machining centers like NC, CNC, DNC, etc. These equipped with supporting activities such as tool changing, automatic work loading/unloading, online process monitoring and control, etc. have significantly enhanced machining capabilities. Such developments in machining have been motivated more by the need to remain competitive in global market. The major effect of these changes can be seen in the increased productivity through substantial reduction of idle time for machines as well as parts. Because of automation and monitoring, the effective operating time - uptime for machines has gone up from 10 % to 65 % (Fig. 1.1). With the improvement in operating time, much more rewards can be reaped from the optimization of machining conditions than with earlier machining environment where uptime itself was quite low. Yet another factor necessitating the optimization of machining conditions is capital intensive investment required by automated machines. Figure 1.2 clearly shows that cost vs cutting speed curves are much steeper for CNC machines than those for conventional machines. Consequently, the optimal cost comes down with the use of NC/CNC machines but the cost of being away from the optimal cutting speed increases rapidly (Fig. 1.2b). Hence it becomes imperative to work at optimal machining conditions as far as possible.

Technical limitations can be critical during machining operations. These limitations usually relate to lack of requisite power and torque, excessive force, bad surface finish, and limitations on speed, feed and depth of cut. As proposed by Lundholm [1991], the machining conditions may be classified as technical, recommended, global economical optimal, and optimal. Their interrelationship is displayed on feed – cutting speed plane in Figure 1.3. The feasible region defined by machining conditions (speed, feed, depth of cut and total number of passes) is bounded by various constraints. A feasible region therefore contains

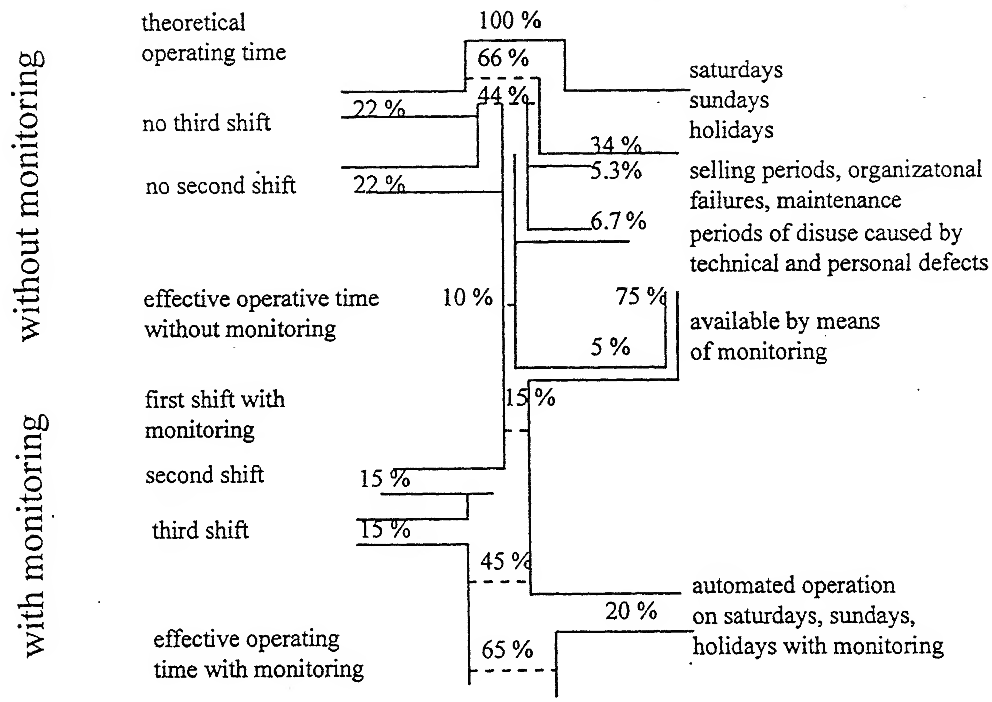

Figure 1.1: Economic importance of monitoring in manufacturing process [Tonshoff et al., 1988].

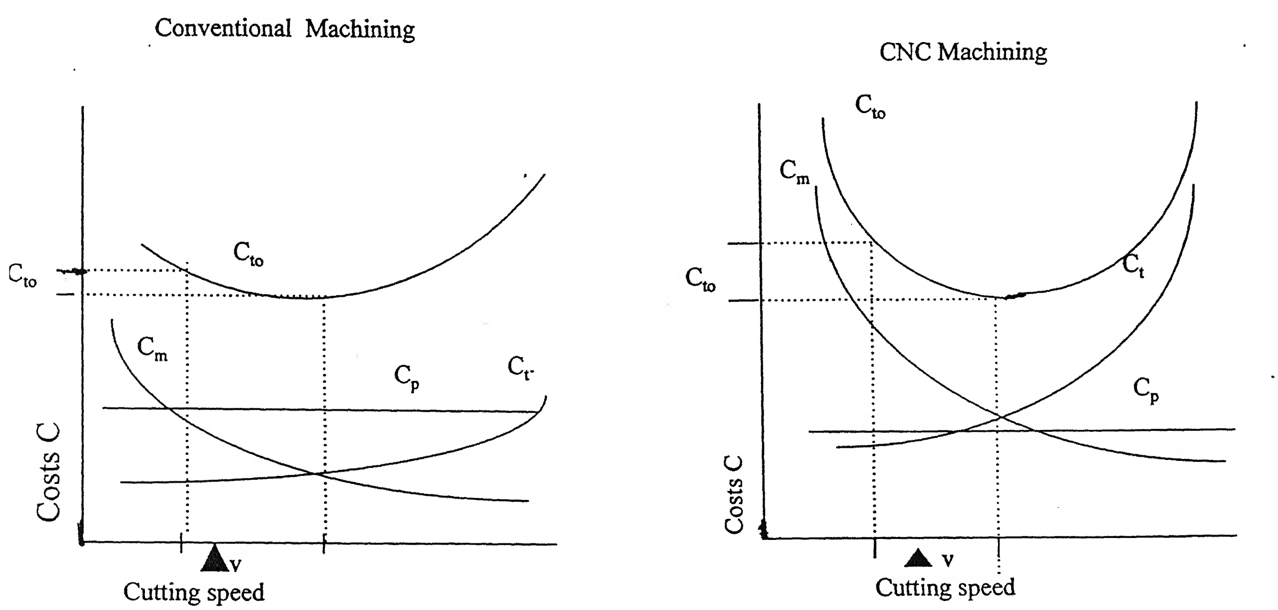

Figure 1.2: Production costs per piece as a function of the cutting speed: (a) conventional machining, (b) CNC machining [Tonshoff et al. 1988].

all possible values of machining conditions satisfying a given set of constraints.

Technical Machining Conditions (TMC)

As shown in Figure 1.3, TMC encompass the whole set of feasible values of machining conditions satisfying all the given constraints in the form of limitations on cutting force and cutting power, surface finish requirements, practical bounds on speed, feed and depth of cut, geometrical and dimensional parameters of the workpiece, and tool and workpiece materials.

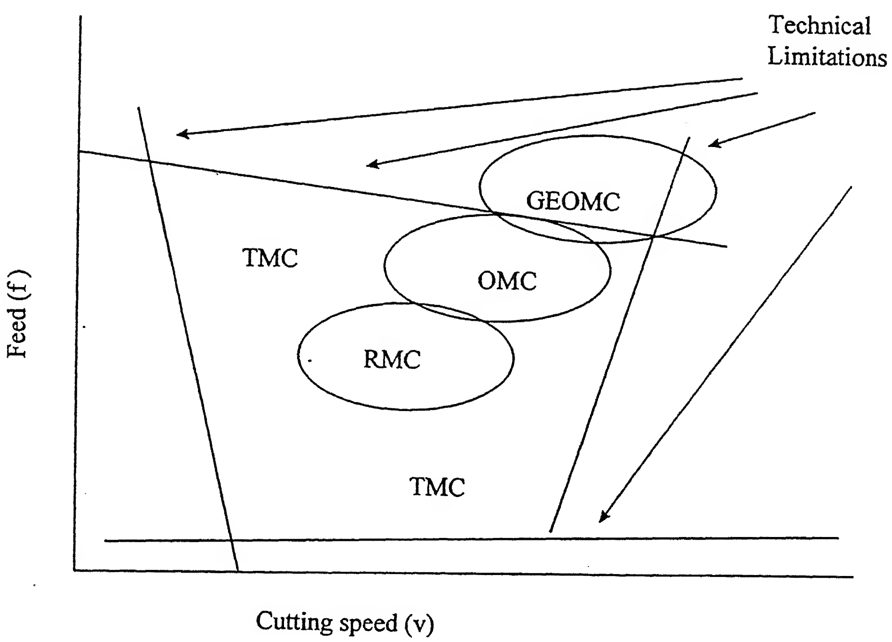

Figure 1.3: Classification of machining conditions [Lundholm, 1991].

Recommended Machining Conditions (RMC)

These are obtained in the form of recommendations from data handbooks [Metcut, 1980; Ai, 1985] which have been compiled over the years on the recommendations of tool suppliers, the experience of manufacturers (users), synthetic analysis of machining processes, and workers' experience. RMC values are invariably conservative. Various expert systems have been developed based on RMC to provide recommended machining conditions. One such system provides the recommendations which are stored in a computer and are retrieved during automated process planning [Wang and Wysk, 1986].

Global Economical Optimal Machining Conditions (GEOMC)

These machining conditions are calculated taking only the economical aspect of the process into account, while completely disregarding the technical limitations. In other words,

GEOMC are determined using unconstrained optimization with the objective comprising various manufacturing costs. Therefore, under many situations, GEOMC lead to unreasonably high values of speed, feed and depth of cut.

Optimal Machining Conditions (OMC)

These are obtained considering economic optimum under technical limitations. OMC, in general, are more favourable than TMC, RMC and GEOMC. These can be determined online as well as offline.

The online determination of OMC requires reliable initial setting of speed, feed and depth of cut. These input machining conditions are optimized online during machining of the workpiece. The online OMC determination involves sensing and monitoring of various events viz. excessive motor current, worn tool, chipping, tool breakage, cutting forces, catastrophic chatter, etc. In order to sense and monitor each of these events separately, or in combination of other events, a number of computer based subsystems are developed. All of these subsystems transmit necessary information to a main controller which is governed by a master computer. An optimization algorithm determines the online OMC after sensing the real time data from various subsystems. The optimal machining conditions thus obtained are supplied to the machining center. This cycle of sensing and monitoring, and optimization of machining conditions is repeated in real-time.

Often all machining conditions cannot be optimized on line. For example in turning operations, the cutting speed and depth of cut are optimized offline and are kept constant during actual machining, whereas the feed is optimized online to obtain OMC. A more useful system shall be that which optimizes both speed as well as feed online keeping depth of cut as constant [Lundholm, 1991].

The offline determination of OMC is performed during planning stage. It may be integrated with machine selection, tool material and geometry selection, and operation sequencing. Some of the recent computer assisted process planning (CAPP) systems have started using mathematical optimization for offline OMC determination. Table 1.1 summarises some of the popular CAPP systems using OMC determined offline. Offline OMC may also act as reliable input to online OMC determination.

1.1.1 Tool Selection

The tool selection essentially means the selection of (i) tool holder and (ii) geometry and material of insert. The selection of tool holder depends on the tool post on one hand and the size and type of insert on the other. Insert selection depends upon the geometrical and dimensional features of blank and final finished part as well as on the machining condition i.e., ranges of speed, feed and depth of cut. The machining conditions, on the other hand, can be determined only when the features of insert are known. This difficulty is, however,

overcome by the fact that tool selection does not require the exact value of machining data but only their ranges [Giusti et al., 1986]. Thus, after selecting the tool the exact machining conditions may be computed considering the economic factors expected to prevail during machining.

Considering the above the tool selection becomes a two stage process. In the first stage a list of tools is generated on the basis of knowledge of cutting technology. It takes into account the problems of geometrical compatibility with workpiece, and differences between roughing and finishing tools. The choice of best tool holder and the insert is not an easy task. It is because the choice criterion often can not be expressed as an exact knowledge. This task gets further complicated because of continuous additions of new tool materials and geometry. An automatic tool selection system should therefore be such that it can be updated without a software specialist. An expert system based on algorithmic if (conditions)-then (action solutions) rules can be a good solution to the problem of tool selection [Guisti et al., 1986]. The other solution method for the first stage is called production rule matrix method and is based on production rules represented in a tabular form instead of logical (if-then) statement form [Domazet, 1990]. This method uses integer multiplication to select a tool and therefore works faster than the expert system. The tool selected at this stage may not be optimal and therefore the second stage is used to find the optimal tool from the list of tools generated at the first stage.

In the second stage, an economic criterion is used to find OMC corresponding to a tool from the list of tools selected in the first stage. The optimal tool from the list of tools may be found using an exhaustive search over a number of tools using the selected criterion. Chen et al. [1989] have presented a heuristic method to automatically select cutting tools for rough turning operations. The selection of tool from the appropriate list of tools is based on an economic objective viz. minimum production cost. The cost of machining with a given tool is estimated after the determination of cutting conditions under constraints. It is possible to estimate, after examination of constraints, whether the next tool in the list will improve cutting conditions and will give reduced minimum production cost or not. The heuristic method works faster and eliminates the need for exhaustive search. The tool selected using the heuristic method may not always be optimal. In case of any difficulty in the implementation of first stage, a list of already available tools in the library may also be used as input to the second stage.

In case only tool material is to be selected (assuming tool geometry to be known), any method to determine OMC can be used and the tool material corresponding to the best value of economic criterion is selected. Lukic et al. [1991] present the Yugoslav approach to solve this problem. This approach makes use of constants of Taylor's tool life equation and is based on some elaborate experimentation. Fenton and Gagon [1993] have plotted the minimum production cost per piece (C_{to}) and minimum production time per piece (t_{to})

points on two axis of a graph for different tool materials. The combination of minimum production cost and minimum production time points which is nearer to the origin is selected and the tool is required to work between these two points.

1.1.2 Tool Life

In a pioneering work, Taylor [1907] suggested a relationship between machining time and tool life for achieving minimum production cost. The relationship has come to be known as Taylor's tool life equation and relates cutting speed (v) and to tool life (T) as

$$vT^{n_1} = C_{T0} \tag{1.1}$$

where n_1 and C_{T0} are constants.

The above equation proposed nearly a century ago is still widely used [Lindstrom, 1989]. The relationship is valid for a given value of feed and depth of cut. An extended form of the above equation which includes the effect of feed (f) and depth of cut (d) is given as [Armerago and Brown, 1969],

$$vT^{n_1} f^{n_2} d^{n_3} = C_T \tag{1.2}$$

Exponents n_1, n_2, n_3 and constant C_T may be evaluated from test data using response surface methodology, ARMA modelling, regression analysis or neural network [Chryssolouris et al., 1990]. The number of tests may be significantly reduced by using random strategy method [Kuljanic, 1980]. Another form of the above equation defining tool life is

$$T = \frac{K_0}{v^{\alpha_0} f^{\beta_0} d^{\gamma_0}}$$

where

$$K_0 = (C_T)^{1/n_1} \; ; \; \alpha_0 = (1/n_1) \; ; \; \beta_0 = (n_2/n_1) \; ; \; \gamma_0 = (n_3/n_1)$$

Colding [1959] suggested a tool life equation in terms of equivalent chip thickness (h_e) and is of the form,

$$vT^{n_4} h_e{}^{n_5} = C_{T1} \tag{1.3}$$

where C_{T1} is a constant and exponent n_4 is a function of v and h_e and exponent n_5 is a function of h_e only. Evaluation of these exponents is not simple, therefore eqn. (1.3) is usually not used for OMC determination. Brewer and Reuda [1963], however, used this equation for OMC determination treating n_4 and n_5 as constants for a given tool-workpiece combination over some working range of v and h_e. Based on the results of machining eight different types of cast irons with three different types of tool materials (HSS, carbide and ceramic), they concluded that exponent n_4 generally depends only on tool material while exponent n_5 depends only on work material. In other words, exponents n_4 and n_5 are constants for a given tool-work material and are independent of v and h_e. The equivalent

chip thickness (h_e) (a ratio of area of cut to contact lenth l_c) is defined as [Brewer and Reuda, 1963]

$$h_e = Gf \tag{1.4}$$

Hence eqn. (1.3) becomes

$$vT^{n_4}(Gf)^{n_5} = C_{T1} \tag{1.5}$$

or

$$vT^{n_4} f^{n_5} G^{n_5} = C_{T1} \tag{1.6}$$

and to a very close accuracy

$$G = \frac{d}{\frac{d-r(1-sin\psi)}{cos\psi} + \pi r \frac{(90-\psi)}{180}} \tag{1.7}$$

Here r and ψ represent the nose radius and side cutting-edge angle of tool, respectively (Fig. 1.4).

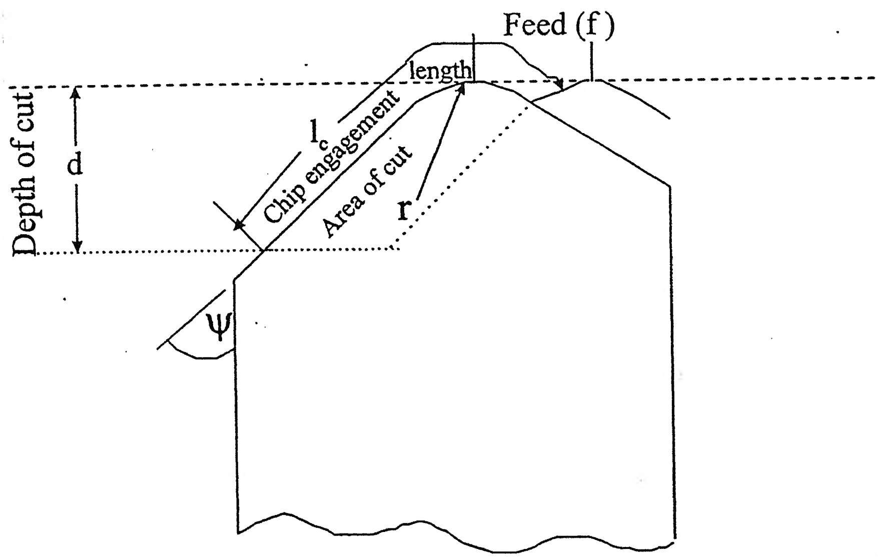

Figure 1.4: Chip equivalent concept [Brewer and Reuda, 1963].

Yellowley and Gunn [1989] have approximated G as

$$G = (d)/(n_6 d + n_7) \tag{1.8}$$

where n_6 and n_7 are constants. Approximating $G = d^{n_8}$ in eqn. (1.8) leads to the extended tool life eqn. (1.2). From eqns. (1.2) and (1.6), it can be easily seen that $n_1 = n_4$; $n_2 = n_5$; $n_3 = (n_8 n_5)$ and $C_T = C_{T1}$.

Exponents n_1, n_2, n_3 may have elements of uncertainty and hence, tool life may turn stochastic. However, it is found that the deterministic models are fairly close to their stochastic counterparts in prescribing OMC. Since eqn. (1.3) has been approximated as eqn. (1.2) and it is widely used for OMC determination. The tool life given by eqn. (1.2) has been used in the present research work. Exponents n_1, n_2 and n_3 have been assumed to be constant, and deterministic tool life equation has been used to construct production cost and production time functions.

1.1.3 Technical Constraints

In real life, turning operation is restricted by various practical constraints in the form of cutting power, cutting force, spindle torque, minimum and maximum tool life, tool thrust, tool wear, maximum tensile stress on tool rake face, maximum temperature in primary shear zone, temperature at chip-tool interface, continuous chip flow requirement, chip breaking region requirement, surface finish and surface integrity requirements, allowable limits of speed, feed, depth of cut, diametral accuracy and any other related constraint. These constraints are represented as functions of machining conditions elements speed, feed and depth of cut. Their functional relationships shall be presented and used in OMC determination in section 2.3 of the next chapter.

1.1.4 Economic Criteria

Machining conditions affect production time and tool life and hence, the production cost. The problem of OMC determination is formulated on the basis of some such criterion. The following two criteria are most commonly used :

1. Minimum production cost: OMC are decided such that cost of producing a workpiece is minimum.

2. Maximum production rate or minimum production time: If the operation is a bottleneck in production line, it is required to be produced in the least possible time.

Apart from these, other criteria in vogue are: (a) maximum tool life, (b) maximum profit rate, (c) minimum of weighted average of production cost and production time, (d) maximum material removal rate and (e) maximum material removal per unit flank wear.

The list is not exhaustive. The criterion of production time per piece, is equivalent to the criterion of production cost per piece. Therefore the method which can be applied to minimization of production cost can also be applied to minimization of production time. The tool life requirement, either on time scale or on the basis of number of pieces produced, may be treated as a constraint during minimization of production cost or production time. Therefore, in the present work, the minimum production cost criterion has been adopted out of the criteria listed above.

1.1.5 Optimization Procedure

Optimization is the act of obtaining best results under given circumstances defined by criterion and constraints. As the effort required and benefit accrued in any practical situation may be expressed as a function of certain variables, optimization is the process which gives maximum or minimum of one or more objective functions subject to a set of constraints and thus guides in the choice of values of decision variables.

Decision Variables

Any optimization problem is conceived with the help of interrelated quantities out of which some are viewed as variables during the design process. In general, certain quantities are fixed at the outset and are called fixed input parameters and some others are treated as decision variables. In the problem of OMC determination for multipass turning, speed, feed, and depth of cut alongwith number of passes, in general, form the set of decision variables and various machining constants and mechanical and metallurgical properties are treated as constants. Sometimes, in order to make the model simple, a few of these decision variables are fixed during the input stage to make them fixed parameters.

Constraints

In many problems, decision variables cannot be chosen arbitrarily; rather they have to satisfy certain specified functional and other requirements. The restrictions which must be satisfied in order to produce an acceptable decision are collectively called decision constraints. The technical constraints described in section 1.1.3 are the constraints in the problem of OMC determination.

Objective Function

Objective function is a measure which guides in the choice of optimal design out of a number of acceptable decisions available. It is a criterion used for comparison of different acceptable decisions for selecting the optimal one. This criterion, when expressed as a mathematically computable function of decision variables is known as objective function. The choice of this function is governed by the nature of problem. The various economic criteria for optimization purpose have been described in section 1.1.4. An objective function can be formulated on the basis of any given criterion. As stated earlier, the criterion used for optimization in the present work is minimum production cost because minimization of production time is similar to minimization of production cost. The tool life requirement has been included as a constraint.

1.2 Literature Survey

The important factors over which the research reported may be distributed are: (a) economic criterion, (b) single or multipass machining, (c) type of model (probabilistic or deterministic), (d) the solution space (unconstrained or constrained; if constrained then type of constraints), (e) optimization approach, (f) solution methodology (use of different assumptions to simplify the model), (g) fixed parameters (d, f and/or n), (h) optimal output decision variables and (i) type of tool life equation (extended Taylor's or modified Colding's). A summary of available literature in the field of optimization of single or multipass turning is presented in Table 1.2. Five different types of models have been used for modelling turning of a cylindrical workpiece. These are discussed in subsequent subsections.

1.2.1 Single Pass Turning Models

This is the basic model for turning. It assumes a constant workpiece diameter and a constant depth of cut. The production cost (C_{to}) or production time (t_{to}) functions when partially differentiated with respect to speed or feed and equated to zero do not give unique feed and speed. According to Armerago and Brown [1969], if either feed or speed is fixed, a point corresponding to minimum production cost or production time can be obtained. There exist a tool life (T_{mc}) for a given depth of cut and feed corresponding to minimum production cost and is obtained as

$$T_{mc} = (\alpha_0 - 1)(\frac{k_l t_c + k_t}{k_l}) \tag{1.9}$$

and the speed (v_{mc}) corresponding to the above tool life T_{mc} is

$$v_{mc} = \frac{K_0^{\frac{1}{\alpha_0}}}{(T_{mc} f^{\beta_0} d^{\gamma_0})^{\frac{1}{\alpha_0}}} \tag{1.10}$$

where k_l, k_t and t_c are over head cost per unit time, tool cost per edge and tool changing time per edge respectively. Similarly tool life (T_{mt}) corresponding to minimum production cost is given by

$$T_{mt} = (\alpha_0 - 1)t_c \tag{1.11}$$

and the speed (v_{mt}) corresponding to the above tool life is

$$v_{mt} = \frac{K_0^{\frac{1}{\alpha_0}}}{(T_{mt} f^{\beta_0} d^{\gamma_0})^{\frac{1}{\alpha_0}}} \tag{1.12}$$

The earlier efforts (Table 1.2) for OMC determination have been mostly limited to the selection of speed using these equations and for a given feed fixed on the basis of experience. In constrained situations if the resulting speed violates any of the constraint then speed is lowered till that constraint is satisfied. If the constraint is not satisfied even at the lowest

possible speed then the next lower feed is selected and again the optimum cutting speed is found using eqn. (1.10) or (1.12) [Brewer and Reuda, 1963; Field et al., 1968].

The research work started with the consideration of all possible combinations of speed and feed and selection of the optimum cutting speed and feed corresponding to minimum production cost or minimum production time for unconstrained turning. Crookal [1969] has solved the single pass turning problem with constraints using performance envelope concept. It is a graphical approach and many graphs have to be plotted in order to find the cutting speed and feed through performance envelope. This method although could not become popular, a good concept viz. production cost or production time per unit turned length has been used for the first time in this work.

The constrained single pass turning model has been solved using different nonlinear programming (NLP) methods viz. Lagrange Function Method [Bhattacharya et al., 1970], Sequential Unconstrained Minimization Technique (SUMT) using Newton Raphson method or Davidon Fletcher and Powell (DFP) method [Iwata et al., 1972; Hati and Rao, 1976], Classical Gradient Method [Elkarmany and Papai, 1978], Neldermead Simplex Method [Agapiou, 1992a & b]. The last method has been used for weighted average of production cost and production time functions.

Since all constraints except the constraints of tool wear (section 2.3.8) and chip-breaking region (section 2.3.12) are monomials, the geometric programming (GP) has been extensively used for minimization of production cost or production time functions derived using extended tool life eqn. (1.2) for single pass turning with constraints. C_{to} or t_{to} function with one constraint constitutes zero degree of difficulty problem (three terms and two variables). Any additional constraint introduced to the problem increases the degree of difficulty by one. Single degree difficulty problem has been solved through maximization of dual problem using differential calculus [Walvekar and Lambert, 1970]. Petropoulus [1973] has solved this problem through numerical iteration. Ermer and Kromodihardjo [1981] have solved the problem of minimum production cost or production time with any number of constraints (a general degree difficulty problem) using GP-LP method [Kochenberger et al. 1973]. The minimization of a posynomial function under posynomial constraints can be performed using this method. However, the rate of convergence for the method becomes slow if more constraints are added [Averiel, 1980]. Eskcioglu [1985] has attempted to solve this Geometric Programming Problem (GPP) by analysis. The two variable problem is solved for all combinations of zero and one degree difficulty problems. The maximum of the dual is selected from all zero and one degree difficulty problems and corresponding primal variables, i.e., speed and feed are found. If this speed-feed combination lies in the feasible range of speed and feed, it is declared as the solution. In fact, it is not the correct solution because the solution which lies in the feasible range of only speed and feed may still lie outside the feasible region defined by all the constraints. Recently Gopalkrishnan and Faiz-AlKhyyal

[1991] have solved one degree difficulty problem by an analytical method. Two zero degree difficulty problems are solved uniquely in dual space and one single degree problem is solved taking intersection point of the two constraints in $ln\ v - ln\ f$ space. The minimum point is located out of the three points using some extra logic. The development of extra logic is not an easy task when there are more than two constraints in single pass turning problems. However, this logic can be developed for such problems because the minima of the primal problem lies either on one constraint or on extreme point of the feasible region defined by intersection of all constraints. The feasible region for monomial constraints is a polygon in $ln\ v$–$ln\ f$ space.

Okushima and Hitomi introduced the maximum profit rate concept in 1964 [Hitomi 1971]. Wu and Ermer [1966] presented a method to determine the maximum profit rate cutting speed for single pass turning case without constraints using the principle of marginal revenue and marginal cost. This model gives the value of optimum speed for a given feed. Armerago and Russel [1966] have analysed the profit rate function for single pass turning, treating speed and feed as variables. Through $(\partial P_r/\partial v) = 0$ and $(\partial P_r/\partial f) = 0$ nomographs they have shown that no unique speed-feed point exists which satisfies these equations. They have also presented the nomographs for different constraints to show how and when these become active. Their attempts have been of theoretical nature only. Hitomi [1971] has presented a computational method to calculate speed while keeping the feed constant. The method has been developed on the basis of differential calculus and the unique combination of speed and feed is obtained using a non-linear programming (NLP) technique. Boothroyd and Rusek [1976] have presented the equation to calculate the tool life (T_{pr}) corresponding to maximum profit rate for a given revenue S and fixed feed and depth of cut as

$$T_{pr} = \frac{1-n_1}{n_1}t_c + \frac{t_p}{k_t S} + \frac{k_t L T_{pr}^{n_1}}{n_1 S v_r T_r} \tag{1.13}$$

where for a reference speed v_r and tool life T_r , the tool life equation is given as

$$v_r T_r^{\ n_1} = C_{T0} \tag{1.14}$$

The speed corresponding to the above tool life (T_{pr}) is obtained as

$$v_{pr} = \frac{C_{T0}}{T_{pr}^{n_1}} \tag{1.15}$$

These equations can be solved numerically and the tool life corresponding to maximum profit rate can be obtained for a given feed. Once tool life is known, the speed corresponding to this tool life can also be obtained. Hati and Rao [1976] have used sequential unconstrained minimization technique (SUMT) alongwith Davidon Fletcher Powell's method (DFP) to maximize profit rate under constraints for single pass turning. Any general non-linear programming method may be applied to this kind of problem. However, GP which

found extensive use in minimization of production cost and production time could not be applied to the maximization of profit rate problem. The reason is that profit rate function is not posynomial [Ermer and Kromodihardjo, 1981]. Gupta et al., [1994] have converted the problem of maximization of profit rate function under posynomial constraints to minimization of a posynomial function under posynomial constraints. It has been achieved by adding few variables and one posynomial constraint to the original problem and the resulting minimization problem is amenable to GP-LP technique.

The knowledge of selling price S is essential for the determination of profit rate and the solution is sensitive to the value of S. Further, it may not be possible to determine the allocation of S for each operation on a workpiece and, therefore, the maximization of profit rate criterion is not commonly used as an economic criterion.

1.2.2 Multipass Turning Models

Multiple passes are adopted in cases where it is not possible to obtain the desired surface finish in a single pass, or it is not possible to remove the total stock, d_{t1}, in a single pass. For multipass turning, the number of passes n_p and the depth of cut for individual passes d_i; $i = 1, \ldots, n_p$ are important decision variables in addition to the speed and feed for individual passes.

The total number of passes n_p includes one finish pass with smaller depth of cut d_s and $n = (n_p - 1)$ number of rough passes such that $n_{min} \leq n \leq n_{max}$. n_{min} and n_{max}, the minimum and maximum possible number of rough passes, can be evaluated as

$$n_{min} = \left[\frac{d_{t1} - d_s}{d_{rmax}}\right]^{+} \tag{1.16}$$

and

$$n_{max} = \left[\frac{d_{t1} - d_s}{d_{rmin}}\right]^{-} \tag{1.17}$$

where d_{rmin} and d_{rmax} are minimum and maximum values for depth of cut in a rough pass, and $[\quad]^{+}$ or $[\quad]^{-}$ imply integer on higher or lower side of the real value of term in brackets. Therefore, the diameter of the workpiece at the start of a pass is not the same for all passes as in multipass turning operation. The models have been generally developed on the basis constant initial or final workpiece diameters ignoring the diameter changes during intermediate passes. The production cost (or production time) per piece in multipass turning is the sum of production costs for individual passes. The production cost based on the initial diameter will therefore be an upper bound on minimum production cost, whereas production cost based on the final diameter will be a lower bound. The same is true for minimum production time function also. Better modelling, therefore, requires consideration of diameter changes during each pass.

Research work in the area of multipass turning may be classified on the basis of selection of depths of cut for all passes. Important selection procedures are: (a) a small depth in the last pass and equal depths in rough passes, (b) unequal depths of cut decided in advance, (c) variable depths of cut and dynamic programming approach, (d) variable number of passes and depths of cut and (e) largest cut and heuristic for turning of stepped shaft.

(a) A Small Depth in the Last Pass and Equal Depths in Rough Passes

A general n_p pass turning problem reduces to n_p single pass turning problems if "small depth in the last pass (to get the desired surface finish) and equal depths in rough passes" strategy is adopted. Further, if constant workpiece diameter assumption is used in place of reduced workpiece diameter after each pass then all rough passes are the same and hence only two (one finish and one rough pass) single pass turning problems have to be solved. Earlier researchers used this approach to solve multipass turning problems. Berra and Barrash [1968] used empirical equations to obtain machining conditions for single pass problems which were not necessarily optimal. Hati and Rao [1976] have found OMC using this approach but have considered reduced diameter after each pass. SUMT alongwith DFP has been used for minimization of production cost for all passes simultaneously. The minimum production cost for all feasible number of passes ranging from n_{min} to n_{max} are obtained and the minimum out of these optimal costs is selected. Thus optimal number of rough passes n^{opt} are obtained. It may be noted that this solution method can handle a finish pass along with n rough passes. The solution will be optimal with respect to d_s only, and overall it is a suboptimal solution.

The upper limit for depth of cut for the finish pass (d_{smax}) is obtained using the relationship [Kals and Hijnk, 1978]

$$d_{smax} = \delta f_{max}(\sin\psi)^2 \tag{1.18}$$

where the slenderness ratio (δ) is defined as ratio of width of cut to undeformed chip thickness and its value is selected such that unmanageable chip flow does not occur. Starting from the upper limit of depth of cut (d_{smax}) and maximum feed (f_{max}) it is checked whether the given feed satisfies all the constraints at a given depth of cut. If all the constraints are not satisfied at this feed value the feed is reduced by a small percentage till all the constraints are satisfied. In case the constraints are not satisfied even at the minimum value of feed (f_{min}) then the depth of cut is reduced by a small percentage and the search starts from the maximum value of feed (f_{max}). In other words, the highest possible depth of cut and the highest possible feed at which all constraints can be satisfied are selected for the finish pass.

Using the constant diameter model, Shin and Joo [1992] have suggested that d_s should be kept at the least possible value d_{smin}. This value of d_{smin} gives the smallest finish pass

production cost. However, it does not guarantee that the overall optimal production cost would result through the least depth of cut selection in the finish pass. They have introduced a constant ζ representing the ratio of tool life for rough pass to finish pass. As the exact value of ζ for multipass turning is not known before hand, introduction of such a constant makes the optimal solution somewhat arbitrary. They have solved an example taking $\zeta = 1$. The salient feature of the model is that it uses tool life as the only parameter on which the total production cost depends. The tool life corresponding to the minimum production cost has been found using a single dimensional Fibonacci's search method. The minimum production cost function is evaluated as sum of one finish pass and n rough passes production costs. The production cost for a particular pass is evaluated using the maximum allowable feed value at that tool life by all constraints for that pass. The speed has been calculated using the optimum tool life and feed values. The model has been used to find the minimum production cost for all feasible number of passes in the range of n_{min} to n_{max}. Thus the optimum number of passes n^{opt} is found. The limitations of the model are:

(i) Selection of d_{smin} restricts the possibility of reducing the number of rough passes by selecting higher depth of cut in the finish pass. Therefore this method sometimes give significantly higher minimum production cost.
(ii) The condition $\zeta = 1$ restricts the selection of the optimum speed, feed, depth of cut and number of passes.
(iii) The model has been made on the basis of initial diameter and therefore does not give accurate minimum production cost.
(iv) All the rough passes are of equal depth.

Van Houtan [1981] uses a graphical search method in speed-feed space for finding single pass OMC. The depth of cut for rough pass has been selected on the basis of equal depth of cut. A CAPP system for rotational parts has been developed using the graphical search method [Van Houtan, 1986] but the model uses constant workpiece diameter. Hayes et al. [1979] have suggested a discrete variable approach to determine OMC for minimum production cost for n rough passes. It is based on exhaustive search in $T - n - n_t$ space, where n_t represents the number of tool changes per part. Although this method has been reported to be very efficient for an example with workpiece length of one meter, it may not be so for small workpiece lengths because the set of feasible n_t contains a large number of elements.

(b) Unequal Depths of Cut Decided in Advance

Ermer and Kromodihardjo [1981] have presented a production cost model for multipass turning based on constant initial workpiece diameter. Depending upon the experience, the

number of passes and the depth of cut distribution among these passes are used as input to the model. A GP–LP technique has been used for the determination of optimum production cost, speed and feed for all passes. The technique can, in principle, handle minimization of a posynomial function under posynomial constraints. Therefore, all the passes can be handled simultaneously.

With the introduction of a few variables and a constraint, the problem of maximization of profit rate function has also been converted to minimization of a posynomial function subjected to posynomial constraints [Gupta et al., 1994]. The only drawback of this kind of OMC determination is in deciding the optimal input number of passes and depth of cut distribution.

Hinduja et al. [1985] have suggested a few rules for the selection or modification of non-uniform depth of cut distribution in rough passes. A term "depth of cut corresponding to minimum production cost" has been used to guide this selection. Hinduja etal., however, did not use this approach in their subsequent papers and adopted equal depth of cut strategy [Chen et al., 1989; Arseculartene et al., 1992]. The salient feature of the above research work is that the satisfactory chip-breaking region is divided into a grid and graphical search is carried out to determine OMC. This ensures easy swarf disposal. On the other side, there is always a possibility of a finer grid and better solution. Further, solution obtained is not optimal.

Yellowley and Gunn [1989] have presented some proof in support of two unequal rough passes approach. According to them two rough passes of equal depth will never give minimum production cost or production time and any greater depth of cut will always lead to better solution. There proof is limited to two pass case only but the same should also hold good for any number of passes. The tool life eqn. (1.6) has been used to reach the conclusion and therefore the same conclusion should hold good for extended Taylor's tool life eqn. (1.2) also.

(c) Variable Depths of Cut and Dynamic Programming Approach

Iwata et al. [1977] have used dynamic programming approach to find the optimal number of passes and depth of cut for each pass. They analyzed the multipass turning problem as a multistage decision making process. At each stage the optimal solution of the previous stage was used as input and a single pass solution method gave the optimum solution for the current stage. The advantage of this approach is that optimum number of passes are uniquely determined as stages required to machine the total depth of cut. The reduced workpiece diameter is made the input to a stage and thus varying workpiece diameter models can be easily solved using DP approach. In this method the total depth of cut is divided into a number of small segments and the optimal distribution of depth of cut is taken as number of segments in each pass. The depth of cut accuracy for a pass will, therefore,

be of the order of the dimension of the segment. Any effort to increase the accuracy would require large number of single pass turning solutions. For example Iwata et al. [1977] have used 10 divisions for a total depth of cut of 10 mm. Hence, the pass can be as accurate as 1 mm. If the depth of cut range for a pass is 1–3 mm, it requires approximately 10 $\times$ 3 = 30 single pass solutions. If the desired accuracy is 0.1 mm then the number of divisions required will be 100 and the number of single pass solutions will increase to 100 $\times$ 21 = 2100. Any nonlinear programming method coupled with dynamic programming approach to solve a multipass turning problem would thus require prohibitive time. Iwata et al. have used SUMT along with Newton Raphson method to solve single pass problems for small number of solutions.

Lambert and Walvekar [1978] have developed a DP approach to solve two pass rough turning problems. GP was used to find single pass solutions. They found that the optimum solution often contains two unequal rough passes. Unklesbey and Crieghton [1978] have simplified this approach and have obtained one finish pass and one rough pass solutions on the basis of constant workpiece diameter. The same rough pass solution is used for any number of rough passes. GP has been used to obtain single pass solutions. Thus the number of single pass solutions has been considerably reduced. The same DP approach has also been used for the determination of optimal number of passes for weighted sum of production cost and production time criteria [Agapiou, 1992 b]. A NLP method (Neldermead simplex method) is used to find single pass solution. This work is very similar to that of Iwata et al. [1977]. The same approach has also been used for obtaining the optimal machining conditions for face milling operation recently [Sonmez et al., 1999].

(d) Variable Number of Passes and Depths of Cut

Since the optimum total number of passes n_p^{opt} are not known in advance, hence it may be treated as a decision variable. Further, the depths of cut for each pass are also unknown at the beginning and therefore these are also decision variables. In this case, the total depth of cut constraint is added to the constraint set.

Chua et al. [1993] have constructed a multipass turning model for rough passes on the basis of initial workpiece diameter for each pass. The objective function (production time per piece) was a polynomial function subjected to one equality constraint in the form of total depth of cut and two monomial constraints for each pass. A NLP method, the sequential quadratic programming (SQP), has been used for optimization. The problem has been solved in two stages. In the first stage n is treated as a continuous variable and a real optimum number of rough passes (n^{optr}) is found on the basis of constant initial workpiece diameter, equal depth of cut for all passes and the same cutting conditions, i.e., the same speed and feed in all passes. In the second stage the same objective function is minimized for $[n^{optr}]^+$ and $[n^{optr}]^-$. This time the model has been made on the basis of changed workpiece

diameter in each passes and with no restrictions on speed, feed and depth of cut for any pass. The minimum out of these two solutions is selected as the solution. This is obviously not an optimal solution to the problem because there is no guideline to the selection of finish depth of cut.

Kee [1996] has suggested that the solution obtained by the above method has high chances of converging to the local minimum at the first stage when n is treated as a continuous variable. Further, there is no evident termination point to the optimal number of passes considered in the search. He has suggested an alternative approach to the solution of multipass rough turning problem. It is based on the study of theoretical economic trends for number of passes and depth of cut distributions, fully utilising the constrained single pass optimization strategy to establish speed and feed for each pass to reach the global optimum n^{opt}. The main features of this work are: (i) it contains a logical method for narrowing down the range of number of passes, i.e., n_{min} to n_{max} to locate the global optimum number of rough passes n^{opt}, (ii) it is based on calculating the production time for multipass turning model with changed workpiece diameter at each pass for a given n^{opt} and depth of cut distribution, and (iii) at the last stage, the depth of cut distribution is refined using a penalty method [Schuldt et al., 1977].
It has been claimed that the above work successfully locates the global optimum number of rough passes. The distribution of depth of cut is obtained using a penalty function method and it may be local minima. In the absence of guidelines for the selection of finish pass depth of cut, the solution obtained is not optimal.

Tan and Creese [1995] have developed a complete model for the calculation of optimal total number of passes (n_p^{opt}) alongwith the speed, feed and depth of cut for each pass. They have made the production cost model on the basis of initial workpiece diameter at each pass for maximum possible number of passes $n_{pmax} = (n_{rmax+1})$ and introduced the same number of binary [0,1] variables. A few constraints have also been added to the original problem with practical constraints. Although the practical constraints are in monomial form, they can assume any other form. The additional constraints ensure that the optimal solution satisfy the binary nature of newly introduced variables while these are treated continuous in the range of zero to one during the solution method. Sequential Linear Programming has been used to obtain the solution for NLP model. A feasible solution is required as the starting point and radius of bound or move limits for each variable are to be defined to start the procedure. The selection of move limits is a trial and error process. The solution method although quite simple (conceptually and numerically) it is not recommended as a technique for general design optimization environment due to lack of robustness and uncertainty in the specification of move limits [Arora, 1989]. It appears to be the only complete general model for multipass turning based on starting diameter of the workpiece at each pass. The value of n_p^{opt} is obtained by counting non-zero binary variables in the optimal solution. Further, the

speed, feed and depth of cut associated with each pass corresponding to non-zero variable is also available with the solution. The model can be solved using any of NLP search methods viz. SUMT with DFP, SQP, etc. This method again gives local minima and has no evident termination point for depth of cut as in the model proposed by Chua et al. [1993].

(e) Largest Cut and Heuristic for Turning of Stepped Shafts

It is an important decision to club machinable volumes horizontally, vertically, or in circular manner and then decide about multiple passes, during rough turning of stepped shafts. The production cost and production time is affected by this grouping of machinable volumes. The recent available research works use a common practice of having the largest possible cut. [Jha, 1996; Prasad et al., 1997]. In a research work related to milling a heuristic based on minimum production cost per unit volume has been used to obtain the minimum production cost [Yellowly and Fisher, 1994]. The solution to the milling problem suggest that sometimes vertical or circular grouping of machinable volumes may lower the production cost.

1.3 Scope and Organization of Thesis

The review of existing literature reveals the following:

1. The problem of determining the optimal machining conditions for turning as well as other machining operations has been drawing attention of various researchers since the beginning of the century. However, much of the literature deals with generalization of models, and adoption and development of newer solution methodology. There appears to be an increasing interest in the determination of optimal machining conditions although some of the processes are quite old.

2. The minimum production cost and minimum production time criteria are most commonly used for the determination of optimal machining condition. Both production cost and production time functions are equivalent from the point of view of optimization. The later can be obtained from the former by neglecting some of the costs and treating others as unity.

3. The optimization problems are formulated on the basis of deterministic as well as probabilistic tool life equations and cost parameters. The deterministic problems are easier to solve and several deterministic model solutions claimed to yield results close to those obtained using stochastic models.

4. Most of the reported works are based on two assumptions: (i) constant initial workpiece diameter and (ii) predetermined uniform/non-uniform depth of cut distribution during

all passes. Depending on the assumptions, different sets of optimal values of speed, feed and depth of cut for each pass are obtained. Further, it may be cumbersome to implement these optimal sets of cutting conditions during manual setting of machining conditions but in the present environment of automated machining with NC/CNC, the setting of different speeds, feeds and depths of cut in various passes would not pose any difficulty.

5. A third assumption of equal tool life in rough as well as finish pass has also been used by some of the researchers to simplify the solution of multipass turning problem with constraints.

6. The majority of technical constraints described in section 2.3, except the constraints of tool wear and effective chip-breaking region, are monomial constraints containing constants and exponents for speed, feed and depth of cut. The estimates of these constants and exponents can be made through experiments. The determination of optimal machining conditions would therefore greatly depend upon the quality of these estimates.

7. There are two types of approaches to find the optimal machining conditions for turning. In the first one, the number of passes, speeds, feeds and depths of cut for all passes are treated as decision variables and their optimal values are obtained from the solution of the model. In the second approach, some of the decisions regarding number of passes and depth of cut distribution are assumed to be known based on some practical knowledge. Their values are given as input to the model and remaining decision variables are obtained from the solution of problem.

8. The various types of optimization methods and algorithms used to obtain optimal machining conditions are as follows:

(i) In a multipass turning problem all the machining conditions including the number of passes, speed, feed and depth of cut for each pass are treated as decision variables. The objective function as well as constraints are linearized about a feasible starting point and linear programming is sequentially applied to better the solution till the optimum is found. The determination of a feasible starting point requires solution of a separate optimization problem and application of sequential linear programming requires careful selection of move limits.

(ii) The multipass turning problem with constraints is converted into unconstrained problem using a penalty function method. The resulting unconstrained problem is solved using SUMT applying any non-linear search method such as Powell's method, Davidon

Fletcher Powell's method or Neldermead Simplex method, etc. These methods require: (a) predetermined value of number of passes, (b) experimentation with a few parameters during SUMT application and (c) possibly a feasible starting point.

(iii) In some of the works the constrained optimization methods viz. generalized reduced gradient (GRG) and sequential quadratic programming (SQP) have been applied for the solution of multipass turning problem with constraints. GRG method requires a feasible starting point, whereas SQP may be started with any arbitrary point and also without any knowledge of parameters for optimization. Therefore, SQP method should be preferred over other methods.

(iv) A special type of nonlinear programming method – geometric programming (GP) has been widely used for single as well as multipass turning optimization where the objective function and the constraints happen to be posynomial. The GP based methods may or may not require a feasible start point. Sometimes the knowledge of depth of cut distribution and number of passes is required to solve the multipass problem.

(v) Another method for solution of multipass turning problem is to break it into several single pass turning problems using predetermined number of passes and depths of cut or use of dynamic programming (DP) which automatically decides the optimal number of passes and depth of cut distribution. The resulting single pass turning problems may be solved using any non-linear programming method, or any other method using differential calculus and graphical search over effective chip breaking region defined in section 2.3. An assumption of equal tool life is used in the method alongwith assumptions of equal workpiece diameter and predetermined uniform or non-uniform depth of cut. The resulting multipass turning problem can be solved using single dimensional search in terms of tool life. At a given tool life, the maximum feed corresponding to the constraints yields the minimum production cost. The method does not require a feasible point. If the three assumptions made above are removed, it turns out to be an easy method than those described in (i) to (iv).

1.3.1 Scope

The minimum production cost or minimum production time criteria are most commonly used for the determination of optimal machining conditions. Both of these criteria are equivalent from the point of optimization. As the later can be derived from the former one, the minimum production cost criterion has been adopted in the present work. The deterministic tool life and cost coefficients have been used to make multipass turning model. As mentioned earlier, there are several approaches to solve multipass turning problem.

These include: (i) linerization of objective function as well as constraints about a feasible initial point and use of linear programming sequentially to better the solution, (ii) conversion of constrained optimization problem to unconstrained optimization one and to solve the resulting unconstrained problem, (iii) use of constrained optimization methods starting from feasible or arbitrary point, (iv) transformation of complex multipass turning problem into several single pass turning problems using assumptions of equal workpiece diameter, equal tool life, predetermined number of passes and uniform or non-uniform depth of cut in each pass, and (v) use of DP to find the optimal number of passes and optimal distribution of depth of cut using single pass solutions.

In the present work, the last two approaches have been considered as they are based on decomposing the complex multipass turning problem into simple single pass turning problem. The available single pass solution methodologies are: (i) nonlinear constrained or unconstrained approach, (ii) geometric programming applicable to posynomial function with posynomial constraints, (iii) graphical methods using differential calculus to search a minima over $(v - d - f)$ space in which the objective function as well as the constraints are described, and (iv) determination of optimal tool life corresponding to minimum production cost using any one dimensional search method and evaluation of minimum production cost by finding the maximum permissible value of feed under the constraints. The last method has been used to obtain single pass solution in the present work because this method works efficiently under the monomial constraints.

The number of passes may be treated as decision variables and the optimal number of passes may be obtained from the solution of optimization problem. This number may also be predetermined based on experience. Two models have been developed in the present work. The first one starts with the maximum number of total passes and finds the optimal number of passes alongwith optimal subdivision of depth of cut among passes. The model has been developed on the assumption of equal (initial) workpiece diameter and equal tool life in all passes. This model yields lower minimum production cost than that obtained using a commonly practised strategy in which the finish pass is made at the lowest possible depth of cut and all rough passes are made of equal depth. The optimal subdivision of depth of cut obtained using this model does not reveal the order in which the depths of cut for rough passes should be used. The second model makes use of dynamic programming and, therefore, varying workpiece diameter model can be handled easily. Further, the optimal number of passes are determined uniquely as stages required to machine the total depth of cut. This model clearly discriminates the order in which the depths of cut should be applied. The assumption of equal tool life in all passes has also been dispensed with. The solution methodology for single pass turning problem has been selected such that once a solution to single pass turning has been obtained for a given depth of cut, the solutions to problems of the same depth of cut with changed workpiece dimensions can be obtained

using parametric analysis. While using dynamic programming to find the optimal number of passes and optimal subdivision of depth of cut among passes, the single pass turning problems with changed workpiece diameter need not to be solved again and again as has been done in the past. The solution to single pass turning problem with changed workpiece diameter can be obtained using simple calculations.

During turning of stepped shafts, various possible sets of machinable volumes and their production costs may be obtained on the basis of horizontal, vertical and circular clubbing of machinable volumes. The minimum production cost of a stepped shaft depends upon the optimal grouping of machinable volumes. There is a need to find the optimal grouping of these machinable volumes using a heuristic method or a mathematical model to obtain minimum production cost. A heuristic based on minimum production cost per unit volume has been suggested to find the optimal grouping of machinable volumes. A mathematical model has also been suggested to obtain optimal grouping of machinable volumes yielding minimum production cost.

1.3.2 Organisation of Thesis

In Chapter 2, the time and cost calculations have been presented to form a production cost function for multipass turning problem. The turning operation is restricted by various practical constraints viz. cutting power, cutting force, etc. and limits on cutting variables. The functional relationships of various constraints in terms of speed, feed and depth of cut have also been presented in this chapter. It may be noted that majority of these constraints are monomials. Finally, Model 1 has been formulated to minimize production cost in multipass turning with monomial practical constraints alongwith total depth of cut constraint for a given number of passes (n_p). The problem is complex without the knowledge of optimal number of passes (n_p^{opt}) and optimal distribution of depth of cut, and due to varying workpiece diameter in each pass. The multipass turning problem with all these constraints can be solved in single stage by considering all the passes simultaneously, or it can be broken into various single pass turning problems using some knowledge to decide the number of passes and depth of cut distribution before optimization. A further simplification has been obtained using the assumption of equal tool life in all passes. Using DP, the multipass turning problem can be viewed as multistage decision making process where each pass constitutes an stage. DP can take into account the varying workpiece diameter and uniquely gives optimal number of passes n_p^{opt} and various d_i for $i = 1, 2 \dots n_p^{opt}$. The main difficulty in the application of DP is that it requires several single pass solutions. Single pass turning or multipass turning problem may be solved alongwith constraints using any NLP method viz. SUMT with Newton Raphson or DFP, GP, GRG and SQP. Some of these methods require a feasible starting point. The determination of feasible starting point requires solution of a series of optimization problems. Therefore, the effort has been

made to select those methods which do not require feasible starting point.

A solution methodology is presented in Chapter 3 for minimization of production cost for single pass turning problem with monomial constraints. The methodology is based on the determination of optimal tool life over working range of tool life using golden section method. It has been observed that the maximum allowable feed under the constraints for a given tool life minimizes the production cost at that tool life if the depth of cut is fixed. In the beginning, attempt has been made to find the optimal number of passes and optimal subdivision of depth of cut among all passes. The assumptions of equal tool life and same initial workpiece diameter in all passes have been used to simplify the problem. A linear integer programming model has been presented to solve the multipass turning problem using LINDO software. It has been observed that optimal tool life, speed, feed and depth of cut depend only on depth of cut under the given constraints for single pass turning. Further, these do not depend on the workpiece diameter or length. The variable component of minimum production cost is directly proportional to workpiece dimensions. This property has been used to find the minimum cost for changed workpiece diameter or length and it significantly reduces the required number of single pass solutions during DP solution of multipass turning problem. The second model described in the same chapter is based on DP. This model successfully determines the optimal number of passes and depth of cut distribution alongwith speed and feed for individual passes.

In Chapter 4 some numerical results have been presented. The first two examples have been solved using integer programming model. It has been found that the commonly practised strategy of using small depth of cut in the finish pass and all rough passes of equal depth of cut does not yield the optimum number of passes or optimum depth of cut distribution. Therefore, the speeds and feeds associated with this solution are also not optimal, whereas integer programming model gives optimal number of passes and optimal depth of cut distribution alongwith speeds and feeds for all passes. It has been found that uneven distribution of depth of cut gives reduced minimum production cost. There are further chances to improve the solution because it is based on assumptions of equal tool life and same workpiece diameter in all passes. The next two examples have been solved using dynamic programming model. It has been found that the tool life at optimum solution is different for finish and rough passes. This model takes into consideration the effect of varying diameter also and provides better results. The last example give comparison of execution time for single as well as multipass turning problem with other nonlinear programming methods viz. SUMT with DFP, GRG and SQP.

In Chapter 5 stepped turning of rough shafts has been considered. Machinable volumes are formed by drawing horizontal and vertical lines from the ends of various cylindrical volumes. Clubbing of machinable volumes horizontally or vertically becomes an important decision to minimize the production cost. The normal practice is to cut the largest possible

length and therefore possibilities of clubbing volumes vertically are eliminated. Vertical clubbing of volumes may reduce the number of passes as well as minimum production cost. A heuristic based on the minimum production cost per unit volume and an integer programming model has been suggested in order to find the optimal clubbing of machinable volumes. The results, in general, show that the heuristic method yields better minimum production cost than that obtained from common practice and the mathematical model gives still better solution.

In Chapter 6 the conclusions derived from the present work have been summarized. It also gives scope for future research as well as possible integration of the present work in CAPP systems for machining.

Sl. No.	System	Researcher(s) [Year]	Obj. Fun.	Pass	Constraints	Soln. Tech.	Machine	Out-puts
1	APPAS	Wysk et al. [1980]	MT	M	M,SF,TH	DC	IBM	v,f
2	CUTPLAN	Zdeblick [1985]	MC	M		DC		v,f, d, *np*
3	TECHTURN	Hinduja et al. [1986]	MC MT	M	CB,CF,DA, P,SF,W	DC d-f		v,f,
4	ROUND	Van-Houtan [1986]	MC	M	CF,DA,P, T,W,SF,TH	DC		v,f, d, *np*
5	ESTPAR	Narang and Fisher [1993]	MC MT	M		GP		v,f d, *np*
6	MPS	Jha, N. K. [1996]	MC	M	P,SF	GP	VAX-8350	v,f d, *np*
7	GIFTS	Prasad et al. [1997]	MT	M	P,SF,DA	GP-LP	PC-386	v,f d, *np*

Table 1.1: Some of the CAPP systems for rotational parts using OMC.

		a	b	c	d	e	f	g	h	i
Sl. No.	Reference	Obj. Fun.	No. of Passes	Type D/P	Cons-traints	Soln. Tech.	Methodology	Input	Output	Tool life
1	Brown [1962]	MC MT	S D	D	U	DC	1	d, f	v	T
2	Brewer and Reuda [1963]	MC	S	D	CF, P	DC	1	d, f	v	C
3	Okushima and Hitomi [1964]	MP	S	D	U	DC	1	d, f	v	T
4	Armerago and Russel [1966]	MP	S	D	CF, P, SF	DC	1	d	v, f	T
5	Wu and Ermer [1966]	MP	S	D	NPC	IT Comp	1	d, f	v	T
6	Berra and Barrash [1968]	MT	M	P	P, SF	EE	2	d, f	v	T
7	Field et al. [1968]	MC	S	D	U	DC	1	d, f	v	T
8	Crookall [1969]	MC MT	S	D	DA, P, SF, W	DC Graph	1	d	v, f	C
9	Walvekar and Lambert [1970]	MC	S	D	P, SF	GP	1	d	v, f	T
10	Bhattacharya et al. [1970]	MC	S	D	P, SF	LFM	1	d	v, f	T
11	Hitomi K. [1971]	MP	S	D	U	DC Comp	1	d f	v	T

Table 1.2 : A summary of literature in the field of optimization of single or multipass turning.

		a	b	c	d	e	f	g	h	I
Sl. No.	Reference	Obj. Fun.	No. of Passes	Type D/P	Cons-traints	Soln. Tech.	Method ology	Input	Output	Tool life
12	Crookall and Ven-katramani [1971]	MT	D	D	P, SF	DC Graph	1	d	v, f	C
13	Rash and Rolstadas [1971]	M(vf)	S	D	SF,	Exhaust Search	1	d	v, f, r	T
14	Iwata et al. [1972]	VMR/ TW	S	P	CF, P, SCR, SF	SUMT-DP-NR	1	d	v, f	T
15	Petropoulos [1973]	MC	S	D	CF, P, SF	GP	1	d	v, f	T
16	Groover [1975]	MC	S	P	SF	DC Simu	1	d, f	v	T
17	Boothroyd and Rusek [1976]	MP	S	D	U	DC	1	d, f	v	T
18	Hati and Rao [1976]	MC MP	M	D/P	CF, P , Th, T	SUMT DFP	2	n, d	v, f	T
19	Iwata et al. [1977]	MC	M	P	CF, P, TW	DP-SU MT-NR	5		v, f, d, np	T
20	El- Karmany and Papai [1978]	MC	S	D	CT, DA, P	CGM	1	d	v, f	T
21	Kals and Hijnk [1978]	MC MT	S	D	CF, DA, DS, SF	DC IT	2		v, f, d ,np	T
22	Lambert and Walvekar[1978]	MC	D	D	P, SF	DP-GP (DC)	2		v, f, d	T

Table 1.2 (continued) : A summary of literature in the field of optimization of single or multipass turning.

		a	b	c	d	e	f	g	h	i
Sl. No.	Reference	Obj. Fun.	No. of Passes	Type D/P	Cons- traints	Soln. Tech.	Method ology	Input	Output	Tool life
23	Sundaram [1978]	GP(d +MT)	S	D	P, SF	GLP	1		v, f, d	T
24	Unklesbay and Creighton [1978]	MC	M	D	CF, P, SF	DP- GP	4		v, f d, np	T
25	Hayes and Davis [1979]	MC	M	D	P, M, SF, T ,TH	Exhaust Search	2		v, f, d, np, nt	T
26	Philipson and Ravindran[1979]	MRR MC	S	D	CF, P, SCR, SF	DC, GP, LP, GLP	1	d	v, f	T
27	Ermer and [1981] Kromodihardjo	MC MT	M	D	P, SF	GP- LP (SP)	3	d	v, f	T
28	Van-Houtan [1981]	MC	M	D	CF, P	DC	2		v, f, d, np	C
29	Hinduja et al. [1985]	MC MT	M	D	CF, DA, P, F, TD, W	DC d-f Search	3		v, f d, np	C
30	Eskcioglu et al. [1985]	MC MT	S	D	DA, P SF, T	GP LFM	1	d	v, f	T
31	Wang and Wysk [1986]		S	D		ES- SR	1	d	v, f	T
32	Yellowley and Gunn [1989]	MC	M	D	M , P, TB	DC	2		v, f d,np	C
33	Gopalkrisnan and Al-khyaal[1991]	MC	S	D	P, SF	GP Ana- lytical	1	d	v, f	T

Table 1.2 (continued): A summary of literature in the field of optimization of single or multipass turning.

		a	b	c	d	e	f	g	h	I
Sl. No.	Reference	Obj. Fun.	No. of Passes	Type D/P	Cons-traints	Soln. Tech.	Method ology	Input	Output	Tool life
34	Agapiou [1992a]	MC+ MT	S	D	CF, P, SF, Th	NMS	1	d	v, f	T
35	Agapiou [1992b]	MC+ MT	M	D	CF, P, SF, Th	DP-SUM T-NMS	5		v, f, d, np	T
36	Arsecularatne et al.[1992]	MC MT	M	D	CF,DA,P SF,W	DC d-f Search	2		v, f d, np	C
37	Jang and Seirag[1992]	VMR	S	D	SF, T, Th,TW	Powell method	1		v, f d	T
38	Jha N.K. [1992]	MC	S	P	P, SF	GP-Sto chastic	1	d	v, f	T
39	Shin and Joo [1992]	MC	M	D	P, SF, T	OPT- T	2		v, f, d, np	T
40	Singh and Raman [1992]		S	D		ES-SR	1		v, f	T
41	Chua et al. [1993]	MT	M	D	P, SF	SQP	5		v, f d, n	T
42	Fenton and Gagon [1993]	MC MP	S	D	U	DC Tra-nsform	1	d, f	v	T
43	Narang and Fisher[1993]	MC	M	D	CF, P, SCR	GP	2		v, f d, np	T
44	Gupta et al. [1995]	MC	M	D	P, SF, T	Opt- T-LIP	4		v, f, d, np	T

Table 1.2 (continued): A summary of literature in the field of optimization of single or multipass turning.

		a	b	c	d	e	f	g	h	I
Sl. No.	Reference	Obj. Fun.	No. of Passes	Type D/P	Cons-traints	Soln. Tech.	Method ology	Input	Output	Tool life
45	Sharper [1995]	MC	S	P	CF, P, SC, SF	Simu-GINO	1	d	v, f	T
46	Tan and Creesse [1995]	MC	M	D	P, SF	SLP	5		v, f d, np	T
47	Chen and Tsai [1996]	MC	M	D	P, T, Th, SF	SA	5		v, f d, np	T
48	Jha N. K. [1996]	MC	M	D	CF, P, SF,	GP	5		v, f d, np	T
49	Kee P. K. [1996]	MC	M	D	CF, P	SUMT-DC Search	5		v, f d, n	T
50	Prasad et al. [1997]	MC	M	D	CF, P	GP-LP (SP)	2		v, f, d, np	T

Table 1.2 (continued): A summary of literature in the field of optimization of single or multipass turning.

Keys for Objective Functions

MC	:	Minimum Production Cost per Piece
MRR	:	Maximum Material Removal Rate
MP	:	Maximum Profit Rate
MT	:	Minimum Production Time per Piece
MC+MT	:	Minimum Weighted Avg. of Production Cost per Piece and Time per Piece
M (vf)	:	Maximum of Product of Speed and Feed
VMR/TW	:	Maximum of Material Removal Volume/Tool Wear
GP (d+MT)	:	Goal Programming (Depth of Cut and MT)

Keys for Number of Passes

S	:	Single Pass Turning
D	:	Double Pass Turning
M	:	Multipass Turning

Key for Model Type

D	:	Deterministic
P	:	Probabilistic

Keys for Constraints

CB	:	Chip - Breaking Region
CF	:	Cutting Force
CT	:	Cutting Time
DA	:	Diametral Accuracy
DS	:	Dynamic Stability
M	:	Torque
NPC	:	Number of Pieces
P	:	Power
SCR	:	Stable Cutting Region
SF	:	Surface Finish
T	:	Tool Life
Th	:	Tool Thrust
TH	:	Temperature at Tool Chip Interface
TW	:	Tool Wear
U	:	Unconstrained Optimization
W	:	Workholding Constraint

Keys for Solution Technique

CGM	:	Conjugate Gradient Methods
DC	:	Differential Calculus
DC Comp	:	DC and Computational
DC Graph	:	DC and Graphical Search
DC IT	:	DC and Iterative

DC d-f	:	DC and Search over d-f Diagram
DC Simu	:	DC and Simulation
DC Trans	:	DC and Transformation
DP-GP	:	Dynamic Programming with GP
DP-GP(DC)	:	Dynamic Programming with GP (using DC)
DP-SUMT-NR	:	DP using SUMT with Newton Raphson
DP-SUMT-NMS	:	DP using SUMT with Neldermead Simplex
EE	:	Empirical Equation
ES-SR	:	Expert System and Storage and Retrieval
LFM	:	Lagrange - Function Method
GLP	:	Goal Programming
Exhaust Search	:	Exhaustive Search
GP	:	Geometric Programming
GP-Analytical	:	GP with Analytical Method
GP-LFM	:	GP with Lagrange's Function Method
GP-LP (SP)	:	GP with Linear Programming using Separable Programming
GP-Stochastic	:	GP with stochastic Programming
IT-Comp	:	Iterative and Computational
LIP-(Opt-T)	:	Linear Integer Programming using Opt-T
Opt-T	:	Optimal Tool Life Search
SA	:	Simulated Anealing
Powell	:	Powell's Method
Simu-GINO	:	Simulation using GINO Package
SLP	:	Sequential Linear Programming
SLQ	:	Sequential Quadratic Programming
SUMT-DFP	:	Sequential Unconstrained Minimization Technique using DFP
SUMT-DC	:	SUMT for Refinement of Results Obtained Search using DC - Search

Key for Methodology

1	:	Single Pass Turning
2	:	Small Depth in Finish Pass and Equal Depth in All Rough Passes
3	:	Depths of Cut are Predecided for All Passes
4	:	Depths of Cut are Determined on the Basis of Equal Workpiece Diameter
5	:	Depths of Cut are Determined on the Basis of Varying Workpiece Diameter

Key for Inputs Parameters

d	:	Depth of Cut
f	:	Feed per Rev.
np	:	Number of Passes

Key of Output Variables

d	:	Depth of Cut
f	:	Feed per Rev.
n	:	Number of Rough Passes
np	:	Total Number of Passes
nt	:	Number of Tool Changes per Part
r	:	Nose Radius
v	:	Speed

Key for Tool Life

C	:	Modified Colding's Tool Life Equation
T	:	Extended Taylor's Tool Life Equation

Chapter 2

Modelling of Turning Operations

2.1 Introduction

The minimum production cost or minimum production time criteria is most commonly used for the determination of optimal machining conditions. The various constraints for a pass are defined in terms of decision variables viz. speed, feed and depth of cut corresponding to each pass. In multipass turning the only constraint connecting all the passes is total depth of cut. The mathematical treatment of optimization problem requires a suitable model and solution methodology to determine the optimal machining conditions viz. the optimal number of passes, speed, feed and depth of cut for each pass. For simplicity, straight turning operation of a cylindrical workpiece has been considered. The problem is to find the expression for production time and production cost and the constraints for single as well as multipass turning. It is assumed that, in general, one final finish pass and a few rough passes are required to complete a job. The corresponding constraints have to be identified and described. The next step will be to formulate the optimization problem and to develop a solution methodology.

Section 2.2 of this chapter deals with the basic model of turning. The expressions for objective functions viz. production time and production cost are derived on the basis of constant as well as varying workpiece diameter. In section 2.3 the description about various constraints is given. The statements of various single as well as multipass turning problems have been presented in section 2.4.

2.2 Objective Function

This section deals with the development of production time and production cost functions for multipass turning of cylindrical bar of initial diameter D_0 and length L in n_p number of passes with last pass as the finish pass. Subscript i is used to specify the pass number which varies from 1 to n_p.

2.2.1 Production Time per Piece (t_{to})

The production time per piece consists of preparation time (t_{pi}), idling time (t_{ai}), machining time (t_{mi}) and tool changing time (t_{ci}) for all passes.

(i) Preparation time (t_{pi}) is the time required to prepare for loading, unloading, etc. for i^{th} pass.

(ii) Idling time (t_{ai}) consists of tool approach and depart time for i^{th} pass and is given as

$$t_{ai} = h_{1i}L + h_{2i} \tag{2.1}$$

where h_{1i} and h_{2i} are constants related to tool approach and depart times. The first term denotes the time required for tool travel from a set point and depends on workpiece length. The second term h_{2i} is the time for engagement and disengagement of tool with workpiece. Idling time consisting of h_{1i} and h_{2i}, in general, is the same for all passes if these passes are machined on the same machine tool. Idling time for various passes will not be the same if these passes are taken on different machine tools.

(iii) Machining time (t_{mi}) is the time needed for actual cutting in i^{th} pass. If the diameter of workpiece at the start of i^{th} pass is (D_{i-1}), then t_{mi} is expressed as

$$t_{mi} = \frac{\pi D_{i-1} L}{1000 v_i f_i} \tag{2.2}$$

where v_i and f_i are speed and feed respectively for i^{th} pass. The intermediate diameters D_{i-1} are calculated as

$$D_{i-1} = D_0 - 2\sum_{k=1}^{k=i-1} d_k \tag{2.3}$$

where d_i is the depth of cut removed in i^{th} pass.

(iv) Tool changing time per tool or per tool tip (t_{ci}) is the time required for tool or tool tip replacement for i^{th} pass. The tool changing time per piece (t_{ei}) for i^{th} pass is defined as

$$t_{ei} = t_{ci}\frac{t_{mi}}{T_i} \tag{2.4}$$

where T_i is the tool life for cutting of i^{th} pass. It is related to the speed, feed and depth of cut by tool life eqn. (1.6) which assumes the following form for a pass i:

$$v_i T_i^{n_{4i}} f_i^{n_{5i}} G_i^{n_{5i}} = C_{T1i} \tag{2.5}$$

Equation (1.6) reduces to extended Taylor's tool life eqn. (1.2) as described earlier. For a specific pass i it is represented as

$$v_i T_i^{n_{1i}} f_i^{n_{2i}} d_i^{n_{3i}} = C_{Ti} \tag{2.6}$$

Hence the total production time t_{to} is

$$t_{to} = \sum_{i=1}^{i=n_p}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p}(t_{mi} + t_{ei}) \tag{2.7}$$

The first two terms of the above expression are constants for a given workpiece length and number of passes. Therefore in actual minimization of production time these may be kept out of objective function and may be added to it after optimization.

2.2.2 Production Cost per Piece (C_{to})

The production cost per piece (C_{to}) consists of material cost, preparation cost, idling cost, machining cost, tool changing cost and tool cost.

(i) Material cost (k_{mc}) is the cost of raw material per piece.

(ii) Preparation cost (t_{pi}) is the cost of loading and unloading per piece for i^{th} pass.

(iii) Idling cost (C_{ai}) is the cost of tool approach and retract per piece for i^{th}pass.

(iv) Machining cost (C_{mi}) is the cost of machining per piece for i^{th} pass.

(v) Tool changing cost (C_{ci}) is the cost of tool or tool tip changing per piece for i^{th} pass.

(vi) Tool cost per piece (C_{ti}) is the cost of tool per piece for i^{th} pass.

In addition, the overhead cost (k_{li}) includes the direct labour and indirect overhead cost per minute for i^{th} pass. The indirect cost is necessary to produce a workpiece and it comprises of depreciation cost of machine tool, general administration expenses, etc. The overhead cost during machining (k_{mi}) includes the cost of cutting oil, electricity charges and any other charges for a specific machine. The cost of tool per edge is represented by k_{ti}. The various cost components (ii) to (vi) can be evaluated as

$$C_{pi} = (k_{li}t_{pi}) \tag{2.8}$$

$$C_{ai} = k_{li}(h_{1i}L + h_{2i}) \tag{2.9}$$

$$C_{mi} = (k_{li} + k_{mi})t_{mi} \tag{2.10}$$

$$C_{ci} = k_{li}t_{ci}(t_{mi}/T_i) \tag{2.11}$$

$$C_{ti} = k_{ti}(t_{mi}/T_i) \tag{2.12}$$

The total preparation cost per piece (C_p), the idling cost per piece (C_a), the machining cost per piece (C_m), the tool changing cost per piece (C_c) and the tool cost per piece (C_t) for multipass turning are expressed as the sum of preparation cost, idling cost, machining cost, tool changing cost and tool cost for individual pass, i.e.,

$$C_p = \sum_{i=1}^{i=n_p} C_{pi} \tag{2.13}$$

$$C_a = \sum_{i=1}^{i=n_p} C_{ai} \tag{2.14}$$

$$C_m = \sum_{i=1}^{i=n_p} C_{mi} \tag{2.15}$$

$$C_c = \sum_{i=1}^{i=n_p} C_{ci} \tag{2.16}$$

$$C_t = \sum_{i=1}^{i=n_p} C_{ti} \tag{2.17}$$

and the total production cost per piece (C_{to}) is

$$C_{to} = k_{mc} + C_p + C_a + C_m + C_c + C_t \tag{2.18}$$

or

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)] \tag{2.19}$$

The first three terms of the cost eqn. (2.19) are constants for a given workpiece length, number of passes and cost elements. Therefore these may be kept out during actual minimization of the production cost function and may be added later.

2.3 Constraints

In the real life, turning operation is restricted by various constraints listed below.

2.3.1 Total Depth of Cut

Let d_{ti} be the remaining depth of cut before a pass i. The total depth of cut to be machined in n_p passes is represented by d_{t1} and it is equal to the sum of depths of cut removed in individual pass, that is,

$$d_{t1} = \sum_{i=1}^{n_p} d_i \tag{2.20}$$

where d_i represents the depth of cut removed in i^{th} pass.

2.3.2 Cutting Force

Total cutting force (F_c) is approximated by the relationship [Chua et al., 1993]

$$F_c = K_1^{'} v^{\alpha_1} f^{\beta_1} d^{\gamma_1} \tag{2.21}$$

where $K_1^{'}$, α_1, β_1 and γ_1 are constants. The value of speeds, feeds and depths of cut for machining should be selected such that the value of cutting force is less than its maximum value (F_{cmax}). Therefore the constraint for cutting force assumes the form

$$K_1 v^{\alpha_1} f^{\beta_1} d^{\gamma_1} \leq 1 \tag{2.22}$$

where $K_1 = K_1^{'}/F_{cmax}$.

2.3.3 Cutting Power

The cutting power (P) is related to machining conditions as [Amerago and Russel, 1966]

$$P = K_2' v^{\alpha_2} f^{\beta_2} d^{\gamma_2} \tag{2.23}$$

where K_2', α_2, β_2, γ_2 are constants. If the maximum available power (P_{max}) is fixed, it is possible to work at speeds, feeds and depths of cut which satisfy the relationship

$$K_2 v^{\alpha_2} f^{\beta_2} d^{\gamma_2} \leq 1 \tag{2.24}$$

where $K_2 = K_2'/P_{max}$.

2.3.4 Torque

The spindle torque (M) is related to decision variables as [Hayes et al., 1979]

$$M = K_3' v^{\alpha_3} f^{\beta_3} d^{\gamma_3} D_0 \tag{2.25}$$

where K_3', α_3, β_3 and γ_3 are constants. The constraint for the torque is expressed as

$$K_3 v^{\alpha_3} f^{\beta_3} d^{\gamma_3} \leq 1 \tag{2.26}$$

where $K_3 = K_3' D_0/M_{max}$; M_{max} being the maximum allowable torque.
The above inequality gives the lower bound on speed, feed and depth of cut because the initial diameter D_0 has been used in the calculation of K_3.

2.3.5 Tool Life

The extended tool life equation is represented as

$$T = \frac{K_0}{v^{\alpha_0} f^{\beta_0} d^{\gamma_0}} \tag{2.27}$$

where

$$K_0 = (C_T)^{1/n_1}; \ \alpha_0 = (1/n_1); \ \beta_0 = (n_2/n_1); \ \gamma_0 = (n_3/n_1)$$

It may be desirable that tool life does not fall below a certain minimum value (T_{min}), i.e.,

$$T \geq T_{min}$$

or

$$K_4 v^{\alpha_0} f^{\beta_0} d^{\gamma_0} \leq 1 \tag{2.28}$$

where $K_4 = T_{min}/K_0$.
In case the tool is engaged for more than one operation at different cutting conditions, the conditions giving the least tool life may be used for restricting minimum tool life. Another

method of restricting tool life is to restrict the minimum number of cuts of equal depth (N_{cmin}) which can be machined in one tool life, i.e.,

$$\frac{Tvf}{\pi D_0 L} \geq N_{cmin} \tag{2.29}$$

or

$$\frac{\pi D_0 L N_{cmin}}{vfT} \leq 1 \tag{2.30}$$

where N_{cmin} is an integer only which can assume the lowest value as one. The initial diameter has been used so that the constraint will be satisfied for all passes.

2.3.6 Tool Thrust

The thrust on tool (F_{th}) is given as [Hayes et al., 1979]

$$F_{th} = K_5^{'} f^{\beta_5} d^{\gamma_5} \tag{2.31}$$

where $K_5^{'}$, β_5 and γ_5 are constants.

In order to avoid tool breakage the thrust on tool should always be less than its maximum allowable value (F_{thmax}), i.e.,

$$K_5 f^{\beta_5} d^{\gamma_5} \leq 1 \tag{2.32}$$

where $K_5 = (K_5^{'}/F_{thmax})$.

2.3.7 Tensile Stress on Rake Face

The breaking of edge of a tool due to brittle fracture is called tool fracture. Fracture occurs when normal stress exceeds the ultimate stress value in uniaxial tension. The point of maximum tensile stress lies on the rake surface at a distance of approximately double the contact length of chip (Fig. 2.1). The tensile stress (σ_t) on tool rake face is given by the relationship [Jang et al., 1992]

$$\sigma_t = K_6^{'} v^{\alpha_6} f^{\beta_6} \tag{2.33}$$

where $K_6^{'}$; α_6 and β_6 are constants.

Therefore the constraint for tool fracture due to stress on rake face is given as

$$K_6 v^{\alpha_6} f^{\beta_6} \leq 1 \tag{2.34}$$

where $K_6 = K_6^{'}/\sigma_{tmax}$; σ_{tmax} being the maximum allowable tensile stress on rake face.

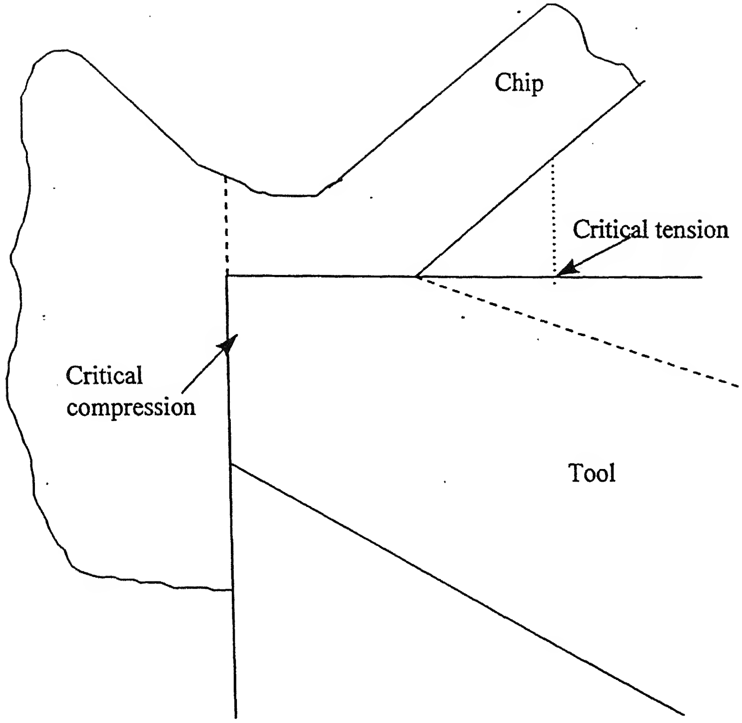

Figure 2.1: Stress distribution in tool [Jang et al., 1992].

2.3.8 Tool Wear

A tool is subjected to two kinds of wear (i) adhesion wear and (ii) diffusion wear. The wear rate of a carbide tool cutting steel due to adhesion (L_{w1}) is given by the relationship [Kannatey-Ashibu, 1985]

$$L_{w1} = K'_{71}(K''_{71} + K'''_{71} v^{\alpha_{71}} {L_w}^{\nu_{71}})^{\delta_{71}} \vartheta^{\alpha_{71}} \tag{2.35}$$

where K'_{71}, K''_{71}, K'''_{71}, α_{71}, ν_{71} and δ_{71} are constants and the wear rate due to diffusion is given by

$$L_{w2} = K'_{72} v^{\alpha_{71}} [exp(-K''_{72}/({K_{72}}''' + {K_{71}}''' v^{\alpha_{71}} {L_w}^{\nu_{71}}))] {L_w}^{\nu_{72}} \tag{2.36}$$

where K'_{72}, K''_{72}, K'''_{72}, ν_{72} are constants and and the permitted flank wear-land value (L_w) is generally taken equal to 0.3 mm [Boothroyd, 1971].

The diffusion wear rate should be lower than the adhesion wear rate to prevent rapid deterioration of the cutting tool, i.e.,[Jang, et al. 1992]

$$L_{w2} \leq L_{w1} \tag{2.37}$$

2.3.9 Primary Shear Zone Temperature

The primary shear zone temperature (θ) should be less than its maximum allowable value (θ_{max}) in order to avoid plastic deformation of tool. Jang [1992] have presented the following relationship for estimating primary shear zone temperature:

$$\theta = K_8^{'} v^{\alpha_8} f^{\beta_8} d^{\gamma_8} \tag{2.38}$$

where $K_8{'}$, α_8, β_8 and γ_8 are constants.
The constraint for maximum primary zone temperature is given as

$$K_8 v^{\alpha_8} f^{\beta_8} d^{\gamma_8} \leq 1 \tag{2.39}$$

where $K_8 = K_8^{'}/\theta_{max}$.

2.3.10 Temperature at Chip-tool Interface

The chip-tool interface temperature (θ_s) dominates at higher cutting speeds. It is considered as another limit on cutting speed. In carbide tools the predominant mechanism of wear is due to crater. In a wide range of working conditions between light and severe craters, the expression for the chip-tool interface temperature is given as [Jang, 1992]

$$\theta_s = K_9^{'} v^{\alpha_9} f^{\beta_9} d^{\gamma_9} \tag{2.40}$$

where α_9, β_9, γ_9, and $K_9^{'}$ are constants.

The chip-tool interface temperature should not exceed the softening temperature of tool (θ_{soften}) otherwise rapid wear initiates, i.e.,

$$K_9 v^{\alpha_9} f^{\beta_9} d^{\gamma_9} \leq 1 \tag{2.41}$$

where $K_9 = (K_9^{'}/\theta_{soften})$.

2.3.11 Stable Cutting Region

In general, it is desirable to have continuous chips with no built-up edge. So that steady-state cutting conditions prevail with no force fluctuations and associated difficulties. The constraint used for this purpose is [Nigm et al., 1976]

$$v^{-\alpha_{10}} f^{-\beta_{10}} \geq K_{10} \tag{2.42}$$

or

$$K_{10} v^{\alpha_{10}} f^{\beta_{10}} \leq 1 \tag{2.43}$$

where K_{10}, α_{10} and β_{10} are constants.

2.3.12 Chip-breaking Region

The chip breaker performs satisfactorily in a given $(d-f)$ region. The information about this region is supplied by the tool manufacturer for a tool insert with a chip breaker groove. A typical $(d-f)$ region for chip breaking region and its approximation is shown in Fig 2.2. The minimum and maximum feed limits (f_{minlim}) and (f_{maxlim}) corresponding to a given

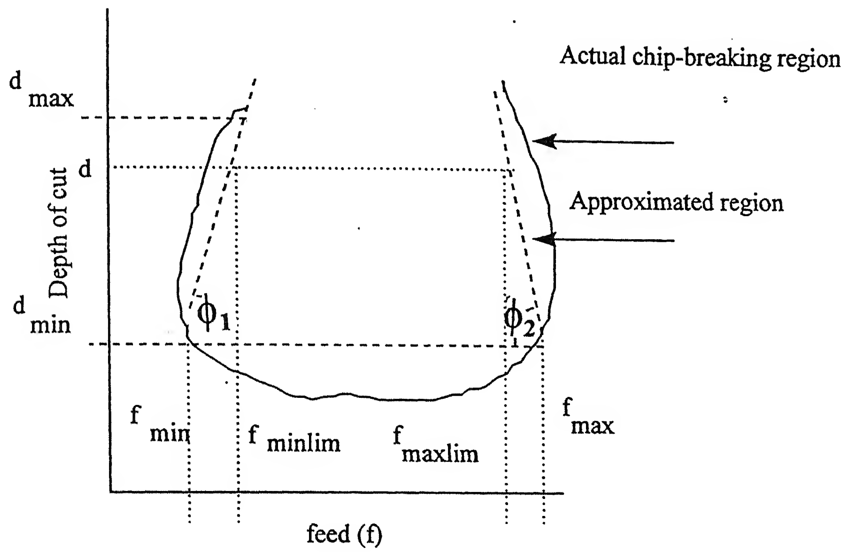

Figure 2.2: Chip-breaking region and its approximation.

depth of cut are obtained from the relationships

$$f_{minlim} = f_{min} + \frac{d - d_{min}}{tan\phi_1} \tag{2.44}$$

and

$$f_{maxlim} = f_{max} - \frac{d - d_{min}}{tan\phi_2} \tag{2.45}$$

Therefore the acceptable feed should lie between feed limits f_{minlim} and f_{maxlim}, i.e.,

$$f_{minlim} \leq f \leq f_{maxlim} \tag{2.46}$$

2.3.13 Surface Finish

The surface finish can be expressed in terms of CLA value (R_a), or as peak-to-valley height (h). The relationship between cutting conditions and surface finish (R_a) as given by Bhattacharya et al. [1970] is

$$K_{12}^{'} v^{\alpha_{12}} f^{\beta_{12}} d^{\gamma_{12}} = R_a \tag{2.47}$$

where $K_{12}', \alpha_{12}, \beta_{12}$, and γ_{12} are constants.

If R_{amax} is the maximum allowable surface roughness value, the constraint for surface finish is given as

$$K_{12} v^{\alpha_{12}} f^{\beta_{12}} d^{\gamma 12} \leq 1 \tag{2.48}$$

where $K_{12} = K_{12}^{'}/R_{amax}$.

A relationship for peak-to-valley height in terms of feed f and nose radius r as presented by Armerago and Brown [1969] is

$$h = \frac{f^2}{8r} \tag{2.49}$$

The peak-to-valley height of the finished surface should be less than its maximum permitted value (h_{max}), i.e,

$$\frac{f^2}{8r} \leq h_{max} \tag{2.50}$$

or

$$f \leq (8rh_{max})^{\frac{1}{2}} \tag{2.51}$$

2.3.14 Surface Integrity

The surface integrity of a surface is quantified by (i) the depth of zone with compressive stress, (ii) the depth of plastically deformed zone, (iii) the value of residual stress and (iv) the compressive stress on the machined surface. Jang [1992] has given the relationship for the estimating the values for (i) to (iv) indicated above.

The depth of zone with compressive stress should be less than its maximum value (d_{czmax}), i.e.,

$$K_{13} v^{\alpha_{13}} f^{\beta_{13}} d^{\gamma_{13}} \leq 1 \tag{2.52}$$

where $K_{13} = (K_{13}^{'} r^{\delta_{13}}/d_{czmax})$ and $K_{13}^{'}$, α_{13}, β_{13}, γ_{13} and δ_{13} are constants.

The depth of plastically deformed zone should be less than its maximum allowable value (d_{pzmax}), i.e.,

$$K_{14} v^{\alpha_{14}} f^{\beta_{14}} d^{\gamma_{14}} \leq 1 \tag{2.53}$$

where $K_{14} = (K'_{14} r^{\delta_{14}}/d_{pzmax})$ and K'_{14}, α_{14}, β_{14}, γ_{14} and δ_{14} are constants.

The residual tensile stress should be less than its maximum allowable value σ_{trmax}, i.e.,

$$K_{15} v^{\alpha_{15}} f^{\beta_{15}} d^{\gamma_{15}} \leq 1 \tag{2.54}$$

where $K_{15} = (K'_{15} r^{\delta_{15}}/\sigma_{trmax})$ and K'_{15}, α_{15}, β_{15}, γ_{15} and δ_{15} are constants.

The residual compressive stress should be less than its maximum allowable value σ_{cmax}, i.e.,

$$K_{16} v^{\alpha_{16}} f^{\beta_{16}} d^{\gamma_{16}} \leq 1 \tag{2.55}$$

where $K_{16} = (K'_{16} r^{\delta_{16}}/\sigma_{cmax})$ and K'_{16}, α_{16}, β_{16}, γ_{16} and δ_{16} are constants.

2.3.15 Diametral Accuracy

A workpiece with length to diameter ratio of more than 6, or a disc with small length to diameter ratio, or a thin-walled section pose certain restrictions during machining. The depth of cut chosen should be such that the radial force causes very little deflection of the workpiece. The diametral accuracy (d_a) should be less than double of maximum deflection (Δ_{max}) [Escikoglu et al., 1985], i.e.,

$$d_a \geq 2\Delta_{max} \tag{2.56}$$

where Δ_{max} is given by

$$\Delta_{max} = \frac{K_{17}' F_r L_o^3}{E D_0^4} \tag{2.57}$$

Then

$$\frac{2K'_{17} F_r L_o^3}{E D_0^4} \leq d_a \tag{2.58}$$

Here E is the modulus of elasticity for work material and F_r is the radial force and its value may be taken as equal to the cutting force F_c. The value of constant K'_{17} depends on chucking and/or holding method such that $K'_{17} = 0.6$ for workpiece held between the chuck and centre, $K'_{17} = 1.4$ for workpiece held between the centres and $K'_{17} = 2.4$ for workpiece held in chuck.

The constraint for diametral accuracy is written as

$$K_{17} v^{\alpha_{17}} f^{\beta_{17}} d^{\gamma_{17}} \leq 1 \tag{2.59}$$

where

$$K_{17} = \frac{2K'_{17} K'_1 L_o^3}{E D_0^4} \tag{2.60}$$

For hollow workpiece $(D_0^4 - D_h^4)$ should be used in place of D_0^4 and $\alpha_{17} = \alpha_1$, $\beta_1 = \beta_1$ and $\gamma_{17} = \gamma_1$.

It may be observed that all the constraints described in this section except the constraint of tool wear and chip-breaking region described in sections (2.3.8) and (2.3.12) respectively are monomials. All the constraints that have been used in literature for the optimization purposes have been summarized above. Several of these constraints are interdependent, e.g., cutting power and torque, tool thrust etc. and torque and any one of them may be included in the process of optimization.

2.4 Formulation of Optimization Problems

In general, the production time per piece function and production cost per piece function are equivalent from optimization point of view. The methodology which is applicable to minimization of production cost may be applied to minimization of production time also. This methodology, however, may not be applicable to maximization of profit rate. The problem of minimization of production cost is presented in this section. The minimization of production time function can also be stated in a similar fashion. The various constraints applicable to the minimization of production cost or time function have already been described in section 2.3. The decision variables for multipass turning problem, as stated before, are the discrete number of passes (n_p), continuous speed (v_i), feed (f_i) and depth of cut (d_i) for each pass i; $i = 1 \ldots . n_p$. The general formulation of multipass turning problem is based on varying workpiece diameter without any restrictions on depth of cut distribution and tool life in any pass.

2.4.1 Varying Workpiece Diameter in All Passes

The workpiece diameter is reduced by an amount equal to twice the depth of cut after each pass. Therefore accurate time and cost results are obtained. In Model 1, the problem has been formulated on the basis of varying workpiece diameter but without any restrictions on depth of cut and tool life in any pass.

Model 1 : Varying Workpiece Diameter without any Restrictions on Depth of Cut and Tool Life in any Pass

The minimization of the production cost (eqn. 2.19) for multipass turning problem with n_p passes is stated as:

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where t_{mi}, T_i and D_{i-1} as given by eqns. (2.2), (1.2) and (2.3) respectively are

$$t_{mi} = \frac{\pi D_{i-1} L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

$$D_{i-1} = D_0 - \lambda \sum_{k=1}^{k=i-1} d_k$$

subject to the following constraints:

(i) The total depth of cut constraint (eqn. 2.20),

$$\sum_{i=1}^{i=n_p} d_i = d_{t1} \tag{2.61}$$

(ii) The total cutting force constraint (eqn. 2.22) for all passes,

$$K_{1i} v_i^{\alpha_{1i}} f_i^{\beta_{1i}} d_i^{\gamma_{1i}} \leq 1 \tag{2.62}$$

(iii) The cutting power constraint (eqn. 2.24) for all passes,

$$K_{2i} v_i^{\alpha_{2i}} f_i^{\beta_{2i}} d_i^{\gamma_{2i}} \leq 1 \tag{2.63}$$

(iv) The spindle torque constraint (eqn. 2.26) for all passes,

$$K_{3i} v_i^{\alpha_{3i}} f_i^{\beta_{3i}} d_i^{\gamma_{3i}} \leq 1 \tag{2.64}$$

(v) The minimum tool life constraint (eqn. 2.28) for all passes,

$$K_{4i} v_i^{\alpha_{0i}} f_i^{\beta_{0i}} d_i^{\gamma_{0i}} \leq 1 \tag{2.65}$$

(vi) The tool thrust constraint (eqn. 2.32) for all passes,

$$K_{5i} f_i^{\beta_{5i}} d_i^{\gamma_{5i}} \leq 1 \tag{2.66}$$

(vii) The tensile stress at rake face constraint (eqn. 2.34) for all passes,

$$K_{6i} v_i^{\alpha_{6i}} f_i^{\beta_{6i}} \leq 1 \tag{2.67}$$

(viii) The tool wear constraint (eqn. 2.37) for all passes,

$$L_{w2i} \leq L_{w1i} \tag{2.68}$$

(ix) The primary shear zone temperature constraint (eqn. 2.39) for all passes,

$$K_{8i} v_i^{\alpha_{8i}} f_i^{\beta_{8i}} d_i^{\gamma_{8i}} \leq 1 \tag{2.69}$$

(x) The chip-tool interface temperature constraint (eqn. 2.41) for all passes,

$$K_{9i} v_i^{\alpha_{9i}} f_i^{\beta_{9i}} d_i^{\gamma_{9i}} \leq 1 \tag{2.70}$$

(xi) The stable cutting region constraint (eqn. 2.43) for all passes,

$$K_{10i} v_i^{\alpha_{10i}} f_i^{\beta_{10i}} \leq 1 \tag{2.71}$$

(xii) The surface finish constraint (eqn. 2.48) or (eqn. 2.51) for all passes,

$$K_{12i} v_i^{\alpha_{12i}} f_i^{\beta_{12i}} d_i^{\gamma_{12i}} \leq 1 \tag{2.72}$$

or

$$f_i \leq (8 r h_{i\ max})^{\frac{1}{2}} \tag{2.73}$$

(xiii) The surface integrity constraints:

(a) The depth of zone with compressive stress (eqn. 2.52) for all passes,

$$K_{13i} v_i^{\alpha_{13i}} f_i^{\beta_{13i}} d_i^{\gamma_{13i}} \leq 1 \tag{2.74}$$

(b) The depth of plastically deformed zone (eqn. 2.53) for all passes,

$$K_{14i} v_i^{\alpha_{14i}} f_i^{\beta_{14i}} d_i^{\gamma_{14i}} \leq 1 \tag{2.75}$$

(c) The residual tensile stress (eqn. 2.54) for all passes,

$$K_{15i} v_i^{\alpha_{15i}} f_i^{\beta_{15i}} d_i^{\gamma_{15i}} \leq 1 \tag{2.76}$$

(d) The residual compressive stress (eqn. 2.55) for all passes,

$$K_{16i} v_i^{\alpha_{16i}} f_i^{\beta_{16i}} d_i^{\gamma_{16i}} \leq 1 \tag{2.77}$$

(xiv) The diametral accuracy constraint (eqn. 2.59) for all passes,

$$K_{17i} v_i^{\alpha_{17i}} f_i^{\beta_{17i}} d_i^{\gamma_{17i}} \leq 1 \tag{2.78}$$

(xv) The efficient chip-breaking region constraint for all passes,

$$f_{minlim\ i} \leq f_i \leq f_{maxlim\ i} \tag{2.79}$$

(xvi) The bounds on decision variables for all passes,

$$v_{i\ min} \leq v_i \leq v_{i\ max} \tag{2.80}$$

$$f_{i\ min} \leq f_i \leq f_{i\ max} \tag{2.81}$$

$$d_{i\ min} \leq d_i \leq d_{i\ max} \tag{2.82}$$

The various constraints related to all passes described by eqns. (2.61) to (2.78) can be represented, in general, as

$$K_{ji} v_i^{\alpha_{ji}} f_i^{\beta_{ji}} d_i^{\gamma_{ji}} \leq 1 \tag{2.83}$$

where $j = 1,2, \ldots, n_c$. The term n_c represents the total number of monomial constraints acting in an individual pass i.

The multipass turning problem can be simplified by making suitable assumptions of equal workpiece diameter in all passes, deciding beforehand the depth of cut distribution among all passes, and keeping tool life to be the same for all passes. The resulting optimization models are given below.

2.4.2 Equal Workpiece Diameter in All Passes

The multipass turning problem is simplified to a great extent by taking initial or some other (mean or final) workpiece diameter value which remains the same in all passes. Initial workpiece diameter D_0 is more commonly used as the fixed diameter for all passes. Model 1 can now be simplified as Model 2 given below.

Model 2 : Constant Initial Workpiece Diameter in All Passes without any Restriction on Depth of Cut and Tool Life

The objective function is same as in model 1, i.e.,

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where t_{mi} is given by eqn. (2.2) after setting $D_{i-1} = D_0$ for all passes and T_i is given by eqn. (1.2). Thus,

$$t_{mi} = \frac{\pi D_0 L}{1000 v_i f_i} \tag{2.84}$$

and

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}} \tag{2.85}$$

subject to constraints (i) to (xvi) of Model 1.

It is important to note that the objective function is now the sum of production costs for individual passes minimized under the constraints for that individual pass, provided the optimal distribution of depth of cut is known. In other words, the total depth of cut constraint is the only constraint which connects the costs of various passes.

2.4.3 Predetermined Depth of Cut Distribution Among All Passes

The earlier solutions of multipass turning problem were based on predetermined number of passes and depth of cut distribution for all passes. A rule may have to be prescribed to select the depth of cut distribution at the outset satisfying the total depth constraint.

Models 1 and 2, with known depth of cut distribution, have been modified for varying and constant workpiece diameter in the following subsections.

Model 3(a) : Predetermined Depth of Cut Distribution with Varying Workpiece Diameter in All Passes

The objective function in this case becomes

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where t_{mi}, T_i and D_{i-1} are given by eqns. (2.2), (1.2) and (2.3), respectively. Now t_{mi}, T_i and D_{i-1} are obtained as

$$t_{mi} = \frac{\pi D_{i-1} L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

$$D_{i-1} = D_0 - 2\sum_{k=1}^{k=i-1} d_k$$

subject to constraints (ii) to (xvi). The total depth of cut constraint has been satisfied during the selection of depth of cut distribution.

Model 3(b) : Predetermined Depth of Cut Distribution with Initial Workpiece Diameter in All Passes

The objective function for this case can be written as

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where

$$t_{mi} = \frac{\pi D_0 L}{1000 v_i f_i}$$

and

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

subject to constraint (ii) to (xvi). The total depth of cut constraint is satisfied during the selection of depth of cut distribution. Models 3(a) or 3(b) can be interpreted as the sum of minimization of n_p number of single pass turning models of the type 4(a) and 4(b) described below.

2.4.4 Single Pass Turning Problems

The multipass turning problem for a known number of passes and depth of cut distribution satisfying the total depth can be broken into a number of single pass turning problems. This is because depth of cut distribution clearly defines the starting workpiece diameters D_{i-1} corresponding to any pass i. The model given below uses varying workpiece diameters for each pass.

Model 4(a) : Single Pass Turning Problem Based on Varying Workpiece Diameter

The objective function for this problem becomes

minimize

$$C_{toi} = k_{li}(t_{pi} + t_{ai}) + [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where

$$t_{mi} = \frac{\pi D_{i-1} L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

$$D_{i-1} = D_0 - 2\sum_{k=1}^{k=i-1} d_k$$

$i = 1$

subject to constraint (ii) to (xvi) for i^{th} pass.

Model 4(b) : Single Pass Turning Problem Based on Initial Workpiece Diameter

In this model the cost function for every pass is derived on the basis of constant initial workpiece diameter and the constraints for each pass are considered separately for the purpose of optimization. The objective function for this case is

minimize

$$C_{toi} = k_{li}(t_{pi} + t_{ai}) + [(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)]$$

where

$$t_{mi} = \frac{\pi D_0 L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

subject to constraints (ii) to (xvi) for i^{th} pass.

2.4.5 Equal Tool Life in All Passes

If the tool life in rough as well as finish pass is the same, the solution of multipass turning problem is further simplified. This assumption is used when all tools are changed after the same period. A one dimensional search method viz. golden section or Fibonacci's method can be now applied to find a single value of tool life for all passes which optimizes production cost for a predetermined depth of cut distribution satisfying the total depth of cut requirement. The minimum of C_{to} function is evaluated using maximum allowable feed for every pass. The maximum feed value for a pass is obtained with the help of constraints for that pass only. A detailed procedure for finding out the maximum allowable feed is presented in section 3.2.

Shin and Joo [1992] have suggested that the least depth of cut should be used in the last finish pass to get minimum production cost in the finish pass and all rough passes should be made of equal depth of cut. They have assumed equal workpiece diameter and equal tool life in all passes.

2.4.6 Optimal Number of Passes (n_p^{opt}) for Model 1

The multipass turning Models 1 to 3 are solved for a given number of passes. The input n_p may or may not be optimal. The methods used to guess the approximate optimal value of number of passes (n_p^{opt}) are given below.

(i) The research work in multipass turning started with the selection of a small depth of cut (d_s) for the finish pass and dividing the remaining depth of cut equally between the rough passes (n). The minimum of the objective function (production cost or production time) corresponding to each value of n is obtained. The minimum of these values is then selected as the minimum of objective function and the corresponding optimum value of rough passes or total passes (n_p^{opt}) is obtained. It may be noted that the solution obtained through this method depends upon the value of depth of cut initially selected for the last finish pass. There is always scope for improvement of the minimum objective function value because an uneven depth of cut can result in improved objective function [Yellowley and Gunn 1989].

(ii) In a slight modification to the above method for selection of optimal number of rough passes, it has been suggested by Shin and Joo [1992] that for the finish pass the depth of cut should be kept at the minimum value d_{smin} to obtain the least production cost in finish pass.

(iii) The above two methods are based on treating the number of rough passes outside the optimization problem. The multipass turning problem has to be solved for every value of number of rough passes ranging from (n_{min}) to (n_{max}), treating the number of rough

passes n as a continuous variable. The cutting parameters viz. speed, feed and depth of cut for all rough passes may be kept equal. Model 2 is now solved on the basis of some d_s value. The resulting real number of rough passes n^{optr} thus obtained guides in the selection of optimal number of passes from Model 1. Model 1 is solved for $[n^{optr}]^-$ and $[n^{optr}]^+$ and without any restriction on the cutting variables for a given value of d_s. This results in uneven distribution of depth of cut for rough passes. The finish pass depth of cut d_s can also be made a decision variable at this stage in order to improve the solution. This part has not been tried in any of the research work reviewed in the present work.

(iv) The solution of the above three methods depend on initial selection of finish pass depth of cut. Therefore, there is need for a method which does not require initial selection of depth of cut for finish pass or the optimal number of passes.

(v) In a recent research work [Tan and Creesse, 1995], the multipass turning model (Model 1) has been redefined for assumed maximum number of passes. A few binary variables corresponding to each pass have been introduced and a few constraints into multipass turning problem have been added. These additional constraints ensure that even though the binary variables are treated as continuous, in the solution they appear only as binary. The optimum number of passes are obtained indirectly by counting non-zero binary variables corresponding to each pass. Another important difference between this problem formulation and Model 1 is that the order of passes is reverse, i.e., $i = 1$ implies the finish pass and $i = 2$ to $i = n_p^{'}$ imply the remaining rough passes. The number $n_p^{'}$ is selected such that it is greater than $(n_{max} + 1)$. The solution of this problem contains the optimal total number of passes n_p^{opt}. The sequential linear programming has been used to find the solution and therefore, any NLP search technique can be used to obtain the solution. The formulation of the general multipass turning problem as proposed by Tan and Creesse [1995] is given below.

Model 5 : Multipass Turning Problem with Varying Workpiece Diameter and without any Restrictions – Assumed Maximum Number of Passes

The objective function for this model can be written as

minimize

$$C_{to} = k_{mc} + \sum_{i-1}^{i=n_p^{'}} m_i k_{li}(t_{pi} + t_{ai}) + \sum_{i-1}^{i=n_p^{'}} m_i[(k_{li} + k_{mi})t_{mi} + (k_{li}t_{ci} + k_{ti})(t_{mi}/T_i)] \tag{2.86}$$

where

$$t_{mi} = \pi \frac{D_{i+1}L}{1000 v_i f_i} \tag{2.87}$$

and

$$D_{i+1} = D_f + 2\sum_{k=1}^{k=i} d_k \tag{2.88}$$

subject to constraints (i) to (xvi) for each pass i; $i = 1, \ldots, n_p'$. Here D_f is the final diameter of the workpiece. The model contains additional binary [0,1] variables m_i corresponding to each pass. First pass or finish pass is denoted by $i = 1$. The last rough pass is denoted by $i = 2$ and so on. The additional constraints are listed below.

The finish pass always exists, i.e.,

$$m_1 = 1 \tag{2.89}$$

Further, if i^{th} pass exists then $(i-1)^{th}$ pass should necessarily exist, i.e.,

$$m_{i-1} - m_i \geq 0 \tag{2.90}$$

The following constraint ensures that variables m_i are binary even if m_i are treated as continuous during optimization, i.e.,

$$\sum_{i=1}^{i=n_p'} m_i d_i = d_{tn_p} \tag{2.91}$$

The zero value of binary variable corresponds to non-existence of that pass. Mathematically, if $m_i = 0$ then $d_i = 0$ and if $m_i = 1$ then $d_i > 0$. These conditions can be combined and written as

$$(d_i/100) - m_i \leq 0 \tag{2.92}$$

where $i = 1, \ldots, n_p'$. Here the depth of cut has been divided by one arbitrary constant 100, assuming the maximum depth of cut in a pass will be less than 100 mm. The value of this constant may be chosen depending upon requirement.

(vi) The complex multipass turning problem is broken into several single pass turning problems. The total depth of cut is divided into a number of equal segments. The depth of cut for each pass is an integer multiple of predetermined segments. The total production cost for multipass turning will be the sum of production cost for all passes. If the multipass turning problem involves n_p passes, they are represented as n_p stages of decision making process. Each single stage problem can be solved using any single pass turning method.

The multipass or multistage turning problem is shown in Figure 2.3. Each stage is represented by a cutting pass. The first $(n_p - 1)$ stages represent rough passes varying from 1 to $(n_p - 1)$ and the last stage n_p represents the finish pass. d_{ti} is defined as a state variable representing the remaining depth of cut before stage i. In other words, d_{t1} represents the

total depth of cut to be removed and $d_{t(n_p+1)}$ is equal to zero since the total depth of cut has been removed in n_p passes. The decision variables are speed, feed and depth of cut for all passes alongwith the number of passes. The minimization of production cost for multipass turning operation can be stated as multistage decision making process as indicated below.

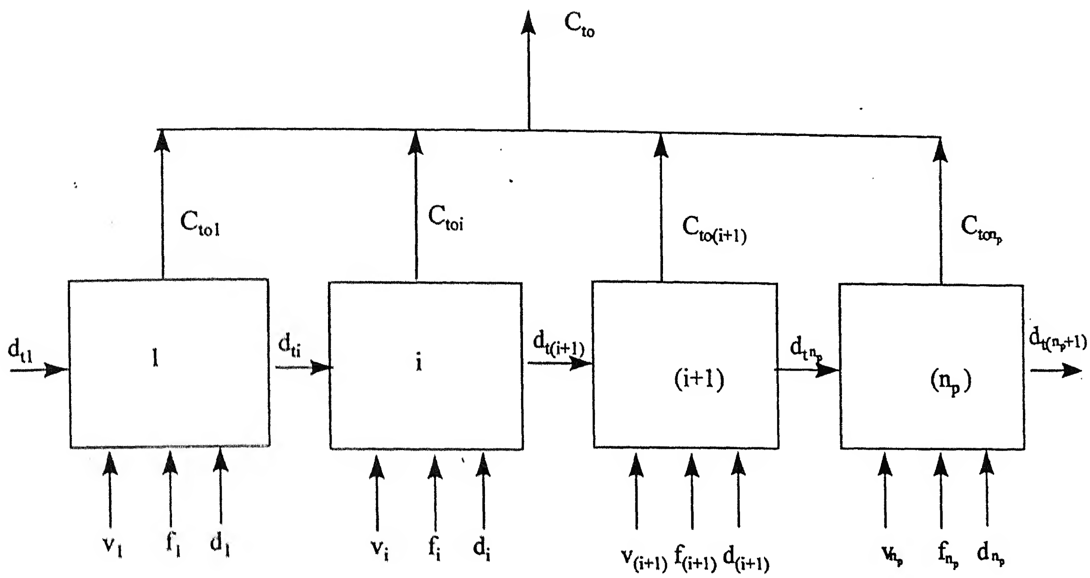

Figure 2.3: Multipass turning described using dynamic programming.

Model 6 : Multipass Turning using Multistage Decision Making Process or Dynamic Programming

For this model the objective function is

minimize

$$C_{to} = \sum_{i=1}^{i=n_p} C_{toi} \tag{2.93}$$

subject to the state transformations

$$d_{t(i+1)} = d_{t(i)} - d_i \tag{2.94}$$

and constraints (ii) to (xvi) for each pass i; $i = 1, \ldots, n_p$.

The total depth of cut constraint has been incorporated in multistage decision making process by the introduction of transformation equations. The production cost $C_{i+1}(d_{t(i+1)})$ at any stage $(i+1)$ is defined as the minimum production cost for stage $(i+1)$ to stage n_p and it is described as

$$C_{i+1}(d_{t(i+1)}) = \text{minimum} \left[\sum_{l=i+1}^{i=n_p} C_{toi}\right] \tag{2.95}$$

Further,

$$C_i(d_{ti}) = \text{minimum}\ [C_{toi} + C_{i+1}(d_{t(i+1)})] \tag{2.96}$$

subject to the constraint (ii) to (xvi).

Now the total depth of cut is divided into a number of equal segments. The process of calculation starts from the last finish pass n_p. The cost $C_{n_p}(d_{tn_p})$ is calculated for various possible values of depth for finish pass using any single pass solution methodology. The past practice has been to optimize these single stage problem each time it is formed [Iwata et al., 1977; Agapiou, 1992b]. Therefore, any effort to increase the number of segments of depth of cut increases the number of single pass solutions. For example, Iwata et al. [1977] have selected 10 divisions for a total depth of cut equal to 10 mm. As mentioned before, the distribution of passes can be as accurate as 1 mm if the depth of cut range is 1 mm to 3 mm and it requires approximately 10 $\times$ 3 = 30 single pass solutions. If the depth of cut distribution is required to be as accurate as 0.1 mm, then 100 segments and approximately 100 $\times$ 21 = 2100 single pass solutions will be required. Any non-linear programming approach coupled with dynamic programming to solve multipass turning problem would thus require prohibitive time.

All the models mentioned above are modified according to actual cutting environment such as the machine tool, cutting tool, practical constraints, bound on decision variables, etc. These may be the same or different for rough passes. However, in the present work it has been assumed that all passes are machined on the same machine tool and all the rough passes are machined with the same type of tool. Although a general solution methodology should consider all these types of constraints, only monomial constraints have been considered alongwith effective chip-breaking region constraint for easy swarf disposal. In the next chapter two models have been presented which give the depth of cut distribution as well as the optimum number of passes. The first model is based on the assumption of equal diameter and equal tool life in all passes and has been formulated as linear integer programming problem, whereas second is based on dynamic programming. The single pass solution methodology has been selected in such a way that single pass problems formed at different stages are not solved each time, instead their solution is obtained using solutions obtained at earlier stages.

Chapter 3

Solution Methodology – Multipass Turning

3.1 Introduction

The various multipass turning problems and associated models have been stated and discussed in Chapter 2. This chapter presents various solution methodologies for multipass turning problems. These have been catagorized on the basis of (i) optimization method, (ii) treatment of number of passes as known parameter or decision variables, (iii) simultaneous treatment of number of passes or synthesising multipass solution using single pass solutions and (iv) remarks.

Table 3.1 contains some of the popular methodologies. The first methodology represents commonly used strategy of using small depth in the finish pass and equal depth in all rough passes. The multipass problem is then solved repeatedly for various feasible values of number of passes to obtain the optimal number of passes. The second methodology aims at automatic selection of number of rough passes by treating them continuous variable [Chua et al., 1993]. The third methodology redefines the multipass turning problem for assumed maximum number of passes. A binary variable corresponding to each pass has been introduced and a few constraints have been added to the problem. These constraints ensure that binary variables may be treated as continuous during optimization and in solution they appear as binary only [Tan and Creesse, 1995]. All the passes are optimized simultaneously using sequential linear programming (SLP) method. The optimal number of passes are obtained by counting non-zero binary variables in the solution. It appears that there is a need for automatic selection of optimal number of passes as well as depth of cut distribution. The fourth methodology is an outcome of the present work. The production costs for a series of feasible depths of cut have been obtained for finish as well as all rough passes. These costs have been derived on the basis of constant initial workpiece diameter and equal tool life in all passes. The multipass turning problem has been reformulated as integer programming problem for assumed maximum number of passes using production costs thus obtained. A

few binary variables and constraints have been added to the new problem. The resulting problem is elaborately described in section 3.4.1 and is solved using LINDO software. The optimal number of passes and associated depth of cut distribution is obtained by counting non-zero binary variables in solution to the integer programming problem.

In the previous chapter the general multipass turning problem has been formulated as multistage decision making model (section 2.4.6). The advantage of multistage decision making or dynamic programming approach is that it uniquely gives the optimal number of passes alongwith the depth of cut distribution required to remove the total depth [Iwata et al., 1977; Agapiou, 1992b]. This approach could not gain popularity because a large number of single pass solutions are required to obtain a multipass solution. This difficulty has been removed in the present work by obtaining solutions for single as well as rough passes for a series of depth of cut only once. At later stages single pass solutions are obtained using simple parametric analysis of the solutions obtained for the first rough pass. Thus single pass problem at the next stage need not be resolved. The last methodology of using mixed variable optimization (discrete number of passes and continuous speed, feed and depth of cut) has not been found in the literature reviewed for multipass turning.

Basically there are two types of approaches to solve multipass turning problems. The first approach optimizes all the passes simultaneously, whereas the second approach synthesizes optimal multipass turning solution using single pass solutions. The present work aims at exploring the later approach. Section 3.2 lists various single pass solution methodologies and describes the selection of a suitable methodology. This is based on selection of optimal tool life corresponding to minimum production cost at a given depth of cut. In section 3.3, a parametric analysis of single pass solution methodology is presented. This analysis helps in finding out minimum production cost for changing workpiece diameter and length for a given depth of cut and constraints without resolving single pass turning problem. The integer programming formulation of multipass turning problem is described in section 3.4.1. Section 3.4.2 describes how the parametric analysis helps in significant reduction in computational efforts for finding single pass solutions during dynamic programming based solution methodology of multipass problems. Three non-linear programming techniques viz. SUMT with DFP, GRG and SQP have been discussed in section 3.5. These shall be compared with DP alongwith single pass methodology in section 4.4.

3.2 Single Pass Turning Methodology

The various solution methodologies available for single pass turning problem are:

(i) Conversion of constrained problem to unconstrained one using penalty function and sequentially minimizing unconstrained problem employing any of the unconstrained methods.

(ii) Solution of constrained problem itself using constrained methods viz. generalized reduced gradient method, sequential linear programming method, sequential quadratic programming method, etc.

(iii) Graphical search over objective function and constraints over $(v-d-f)$ space using differential calculus and some logic based on constants and exponents of tool life equation and constraints.

(iv) Use of geometric programming based methods which started around 1970. These are still preferred due to posynomial objective function and monomial constraints [Narang and Fisher, 1993; Jha, 1996; Prasad et al., 1997; Sonmez et al., 1999].

(v) Classical application of differential calculus where optimal tool life is obtained and the corresponding speed is found for a fixed feed and depth of cut in a single pass turning.

In constrained turning problem, where all the constraints are monomial, the optimal tool life is obtained using one dimensional search. During one dimensional search the minimum production cost is found using maximum allowable feed under the constraints at a given tool life and depth of cut. Once optimal tool life and corresponding maximum feed have been obtained, the optimal speed is found using tool life equation.

The last method has been used to obtain minimum production cost because it does not require differentiation, expensive computation viz. matrix inversion or solution of non-linear equations, etc. Further it does not require determination of initial feasible starting point which may be a necessary requirement for some of the methods described in (i) to (iv). Therefore this method has been adopted in the present work and the results discussed in the next chapter will justify its selection. The single pass turning model described in section 2.4.4 with monomial constraints and effective chip-breaking region constraint alongwith bounds on variables is given as:

minimize

$$C_{toi} = (t_{pi} + t_{ai}) + K0_{1i} t_{mi} + K0_{2i} \frac{t_{mi}}{T_i} \tag{3.1}$$

where

$$t_{mi} = \frac{\pi D_0 L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

$$K0_{1,i} = k_{li} + k_{mi}$$

$$K0_{2,i} = k_{li} t_{ci} + k_{ti}$$

subject to: (i) n_c number of monomial constraints,

$$K_{ji} v_i^{\alpha_{ji}} f_i^{\beta_{ji}} d_i^{\gamma_{ji}} \leq 1$$

(ii) effective chip-breaking constraint,

$$f_{minlim\ i} \leq f_i \leq f_{maxlim\ i}$$

(iii) bounds on decision variables,

$$v_{i\ min} \leq v_i \leq v_{i\ max}$$

$$f_{i\ min} \leq f_i \leq f_{i\ max}$$

$$d_{i\ min} \leq d_i \leq d_{i\ max}$$

Here for a given i^{th} pass, $j = 1 \ldots n_c$.
Using extended tool life eqn. (1.2), v_i is defined as

$$v_i = \frac{C_T}{T_i^{n_{1i}} f_i^{n_{2i}} d_i^{n_{3i}}} \tag{3.2}$$

and the objective function (3.1) is redefined as

$$C_{toi} = A0_{1i} d_i^{n_{3i}} f_i^{-(1-n_{2i})} + A0_{2i} \tag{3.3}$$

where

$$A0_{1i} = K0_{1i}\left(\frac{\pi D_0 L}{1000 T_i}\right)\left(\frac{T_i^{n_{1i}}}{C_T^{n_{1i}}}\right)\left(T_i + \frac{K0_{2i}}{K0_{1i}}\right) \tag{3.4}$$

$$A0_{2i} = K0_{1i}(t_{pi} + t_{ai}) \tag{3.5}$$

It may be noted that $A0_{1i}$ is a constant at a given tool life T_i and the exponents $(1 - n_{2i})$ and n_{3i} are always positive. Therefore the production cost for a pass is minimized by using the minimum permissible value of depth of cut and maximum permissible value of feed by the constraints, provided the tool life remains same.

Using the above clue the minimum of objective function eqn. (3.1) is determined at various test points along tool life between the minimum and maximum tool life limits $(T_{i\ min})$ and $(T_{i\ max})$. These limits are obtained using the relationships

$$T_{i\ min} = \frac{C_T^{\frac{1}{n_{1i}}}}{v_{i\ max}^{\frac{1}{n_{1i}}} f_{i\ max}^{\frac{n_{2i}}{n_{1i}}} d_{i\ max}^{\frac{n_{3i}}{n_{1i}}}} \tag{3.6}$$

and

$$T_{i\ max} = \frac{C_T^{\frac{1}{n_{1i}}}}{v_{i\ min}^{\frac{1}{n_{1i}}} f_{i\ min}^{\frac{n_{2i}}{n_{1i}}} d_{i\ min}^{\frac{n_{3i}}{n_{1i}}}} \tag{3.7}$$

Using $\alpha_{0i} = \frac{1}{n_{1i}}$; $\beta_{0i} = \frac{n_{2i}}{n_{1i}}$; $\gamma_{0,i} = \frac{n_{3i}}{n_{1i}}$ and $K_{0i} = C_T^{\frac{1}{n_{1i}}}$, the tool life has also been defined as eqn. (1.27), i.e.,

$$T_i = \frac{K_{0i}}{v_i^{\alpha_{0i}} f_i^{\beta_{0I}} d_i^{\gamma_{0i}}} \tag{3.8}$$

Equations (3.5) and (3.6) can now be rewritten as

$$T_{i\ min} = \frac{K_{0i}}{v_{i\ max}^{\alpha_{0i}} f_{i\ max}^{\beta_{0i}} d_{i\ max}^{\gamma_{0i}}} \tag{3.9}$$

$$T_{i\ max} = \frac{K_{0i}}{v_{i\ min}^{\alpha_{0i}} f_{i\ min}^{\beta_{0i}} d_{i\ min}^{\gamma_{0i}}} \tag{3.10}$$

Any one dimensional search method viz. golden section or Fibonacci's method can be used for selection of test points. The minimum of minimum objective function value at test points is selected as the optimum production cost at a given depth of cut. The corresponding optimum cutting parameters for that depth of cut d_i are obtained using the steps given below.

Step 1

The maximum permissible feed for a given j^{th} constraint (f_{ji}^*) is obtained from

$$(f_{ji}^*)^{\beta_{ji}} = \frac{1}{K_{ji} v_i^{\alpha_{ji}} d_i^{\gamma_{ji}}} \tag{3.11}$$

substituting for v_i from eqn. (3.7),

$$(f_{ji}^*) = \left(\frac{T_i^{\alpha_{ji}}}{K_{0i}^{\frac{\alpha_{ji}}{\alpha_{0i}}} K_{ji} d_i^{exp1}}\right)^{\frac{1}{exp2}} \tag{3.12}$$

where $exp1 = \gamma_{ji} - \gamma_{0i}\frac{\alpha_{ji}}{\alpha_{0i}}$, $exp2 = \beta_{ji} - \beta_{0i}\frac{\alpha_{ji}}{\alpha_{0i}}$. The $exp2$ is nonzero quantity otherwise the constraint does not limit the permissible value of feed.

Step 2

The maximum permissible feed for i^{th} pass (f_i^*) is obtained as

$$f_i^* = min\ (f_{ji}^*) \quad \text{for all } j$$

Step 3

The minimum cost for i^{th} pass at the test point is obtained from

$$C_{toi}^* = A0_{1i} d_i^{\frac{\gamma_{0i}}{\alpha_{0i}}} (f_i^*)^{-(1-\frac{\beta_{0i}}{\alpha_{0i}})} + A0_{2i} \tag{3.13}$$

Step 4

At a given test point T_i the cutting speed v_i^* corresponding to minimum production cost is obtained by the following relationship:

$$v_i^* = \frac{K_{0i}^{\frac{1}{\alpha_{0i}}}}{T_i (f_i^*)^{\frac{\beta_{0i}}{\alpha_{0i}}} d_i^{\frac{\gamma_{0i}}{\alpha_{0i}}}} \tag{3.14}$$

Step 5

The optimum tool life for i^{th} pass (T_i^*) is located as the result of one dimensional search. The corresponding maximum permissible feed value (f_i^*) and optimum cost (C_{toi}^*) are also obtained.

3.3 Parametric Analysis

The multipass turning model is accurately defined using varying workpiece diameter at each pass instead of constant initial workpiece diameter. Therefore, the solution to single pass turning problem with varying diameter for a given depth of cut and cutting environment is repeatedly required. The minimization of production cost eqn. (3.1) under the constraints has been obtained by selecting optimal tool life using one dimensional search method. The minimum production cost at a test point T_i is obtained by selecting the maximum permissible feed under the constraints. The optimal cutting speed corresponding to a given depth of cut is found from the consideration of optimal tool life T_i^* at a given depth of cut. Thus, minimum production cost at a given depth of cut is function of T_i^* only. This optimal tool life T_i^* remains the same for changed workpiece dimensions. The practice has so far been to formulate a new problem each time the workpiece dimensions are changed and solve the new problem [Iwata et al., 1977; Agapiou, 1992b; Jha, 1996]. The cost eqn. (3.3) is given as

$$C_{toi} = A0_{1i} d_i^{n_{3i}} f_i^{-(1-n_{2i})} + A0_{2i} \tag{3.15}$$

where

$$A0_{1i} = K0_{1i} \left(\frac{\pi D_0 L}{1000 T_i}\right) \left(\frac{T_i^{n_{1i}}}{C_T^{n_{1i}}}\right) \left(T_i + \frac{K0_{2i}}{K0_{1i}}\right) \tag{3.16}$$

and

$$A0_{2i} = K0_{1i}(t_{pi} + t_{ai}) \tag{3.17}$$

$A0_{2i}$ is independent of the decision variables and $t_{ai} = h_{1i}L + h_{2i}$. Therefore,

$$A0_{2i} = K0_{1i}(t_{pi} + h_{1i}L + h_{2i}) \tag{3.18}$$

During optimization this constant can be evaluated separately and only the first term of eqn. (3.3) is to be minimized.

3.3.1 Single Pass Turning with Varying Workpiece Dimensions

The multipass turning model is accurately defined using varying workpiece diameters for each pass. The objective function in Model 4(a) therefore makes use of varying workpiece diameter instead of original workpiece diameter. It has been a practice to solve the single pass turning problem separately for all intermediate workpiece diameters even when the depth of cut remains the same in these cuts [Iwata et al., 1977; Agapiou, 1992b].

In this section a simple interpretation of optimum production cost obtained in the last section is presented. It may be noted that optimum tool life and hence optimum cutting parameters v_i^* and f_i^* do not change with changed workpiece dimensions (diameter and length) for the same depth of cut and under the same cutting environment. Further, the new optimum production cost can be obtained using the previous results. The production cost given as

$$C_{toi}^* = A0_{1i} d_i^{n_{3i}} (f_i^*)^{-(1-n_{2i})} + A0_{2i} \tag{3.19}$$

may be redefined as

$$C_{toi}^* = A0'_{1i} D_i d_i^{n_{3i}} (f_i^*)^{-(1-n_{2i})} + A0_{2i} \tag{3.20}$$

$$A0'_{1i} = \frac{A0_{1i}}{D_0}$$

or

$$C_{toi}^* = A0''_{1i} D_i d_i^{n_{3i}} (f_i^*)^{-(1-n_{2i})} + A0_{2,i} \tag{3.21}$$

$$A0''_{1i} = \frac{A0_{1i}}{D_0 L}$$

It may be noted that effort has always been to find f_i^* under constraints at a given test point to minimize the production cost. Constant $A0'_{1i}$ or $A0''_{1i}$ only depend on the tool life at test point T_i. Further, these are independent of the workpiece dimensions. The optimum production cost for subsequent pass or changed workpiece dimensions can be obtained using eqn. (3.20) or eqn. (3.21) and the results of previous pass, provided depth of cut and cutting environment remain the same. The value of constant $A0_{2i}$ is calculated using eqn. (3.18) when change in workpiece length takes place. In actual application the first term of eqn. (3.19) is a variable cost. The maximum allowable feed f_i^* under the constraints for a given depth of cut is also constant. Therefore, the optimal variable cost $C_{tovar\ i}^*$ corresponding to given depth of cut is

$$C_{tovar\ i}^* = A0_{1i} d_i^{n_{3i}} (f_i^*)^{-(1-n_{2i})} \tag{3.22}$$

The variable cost per unit diameter corresponding to given depth of cut is

$$C_{tovar\ i}^{*\prime} = \frac{C_{tovar\ i}^*}{D_0} \tag{3.23}$$

and the variable cost per unit depth and unit length is given as

$$C_{tovar\ i}^{*\prime\prime} = \frac{C_{tovar\ i}^*}{D_0 L} \tag{3.24}$$

These conclusions are valid for any test point T_i and are, therefore, also applicable to the optimal tool life T_i^* in pass i. In that case the constants $A0_{1i}$ and $A0_{2i}$ are to be calculated using T_i^*.

3.4 Multipass Turning Models

The multipass turning model described in this section with monomial constraints and effective chip-breaking region constraint alongwith bounds on variables is given as

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi} + t_{ai}) + \sum_{i=1}^{i=n_p} K0_{1i} t_{mi} + \sum_{i=1}^{i=n_p} K0_{2i} \frac{t_{mi}}{T_i} \tag{3.25}$$

where

$$t_{mi} = \frac{\pi D_{i-1} L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_{1i}}}}{v_i^{\frac{1}{n_{1i}}} f_i^{\frac{n_{2i}}{n_{1i}}} d_i^{\frac{n_{3i}}{n_{1i}}}}$$

$$K0_{1i} = k_{li} + k_{mi}$$

$$K0_{2,i} = k_{li} t_{ci} + k_{ti}$$

subject to:

(i) total depth of cut constraint,

$$\sum_{i=1}^{i=n_p} d_i = d_{t1}$$

(ii) n_c number of monomial constraints,

$$K_{ji} v_i^{\alpha_{ji}} f_i^{\beta_{ji}} d_i^{\gamma_{ji}} \leq 1$$

(iii) effective chip-breaking constraint,

$$f_{minlim\ i} \leq f_i \leq f_{maxlim\ i}$$

(iv) bounds on decision variables

$$v_{ifmin} \leq v_i \leq v_{ifmax}$$

$$f_{imin} \leq f_i \leq f_{ifmax}$$

$$d_{ifmin} \leq d_i \leq d_{ifmax}$$

where $i = 1 \dots n_p$ and $j = 1 \dots n_c$.

The multipass turning problem, in general, has the following design variables: (i) the optimum number of passes (n_p^{opt}), (ii) the optimal depth of cut distribution among the passes and (iii) speeds and feeds for all passes. In order to find the solution to the problem, the assumptions viz. constant initial workpiece diameter, predetermined depth of cut distribution, and equal tool life for all passes are used. These assumptions make the solution of the problem easy but at the cost of accuracy. In the present work, an attempt has been made to develop two models in which these assumptions have been removed in steps. An integer programming model is proposed that is based on constant initial workpiece diameter and equal tool life. The second model is based on dynamic programming approach without the above assumptions.

3.4.1 An Integer Programming Model

This model is based on constant initial workpiece diameter in all passes. It has also been assumed that all the rough as well as finish pass tools are changed simultaneously. Also, instead of using a rule to define the depth of cut distribution satisfying total depth of cut requirement, an optimum depth of cut distribution alongwith optimal number of passes has been found using integer programming.

The total production cost minimization is achieved in two phases. In the first phase, separate minimum costs for individual rough pass and finish pass are determined and tabulated for various fixed values of depth of cut. The values of depth of cut are selected from a series of depths. In the second phase, optimal subdivision of depth of cut for rough passes and finish pass, optimal number of passes and minimum total production cost are determined using an integer programming model.

Phase 1

This phase consists of determining costs for individual finish or rough pass considering various fixed values of depth of cut. A series for depth of cut is defined. The q^{th} element of the series, for i^{th} pass, d_{iq} is $d_{iq} = d_{i\ min} + (d_{i\ max} - d_{i\ min})(\frac{q}{m_i})$; $q = 0, 1, 2 \dots m_i$, where m_i is a suitable integer. The value of m_i is selected considering $d_{i\ min}$, $d_{i\ max}$ and the least count available on the machine tool for adjusting the depth of cut.

Phase 2

In this phase, an optimal combination of depths of cut d_{0q}^{opt} and d_{iq}^{opt}, $i = 1, 2 \dots n$, optimum number of rough passes n^{opt}, and total minimum production cost C_{to}^{opt} are determined. Initially a large value of number of rough passes is taken. The number of passes needed to

remove the total depth of material will be $(n+1)$, including the finish pass. The integer variables $X_{iq}; i = 0, 1, 2...n; q = 1, 2..m_i$ are defined such that

$i = 0$ implies finish pass

$i = 1, 2..n$; implies i^{th} rough pass

$q = 1, 2...m_i$; implies correspondence to q^{th} depth of cut

$X_{iq} = 1$ if d_{iq} value of depth of cut is selected in i^{th} pass,

$= 0$ if d_{iq} value of depth of cut is not selected in i^{th} pass.

Minimization of total production cost C_{to} may be stated as the following integer programming problem:

minimize

$$C_{to} = \sum_{i=0}^{n} C_{toi} \tag{3.26}$$

Alternatively, the problem can be stated as

minimize

$$C_{to} = \sum_{i=0}^{n} \sum_{q=0}^{m_i} C_{iq} X_{iq} \tag{3.27}$$

where corresponding to a particular d_{iq}, $C_{iq} = C_{toi}$ for $i = 0, 1, 2..n$.

subject to the constraints

$$\sum_{i=0}^{m_0} X_{0q} = 1 \text{for i = 1,2....n} \tag{3.28}$$

$$\sum_{q=0}^{m_i} X_{iq} \leq 1 \tag{3.29}$$

and

$$\sum_{i=0}^{n} \sum_{q=0}^{m_i} d_{iq} X_{iq} = d_{t1} \tag{3.30}$$

Constraint eqn. (3.28) implies that there is only one d_{0q} selection for the finish pass and the finish pass must always be selected. Constraint eqn. (3.29) implies that there is only one d_{iq} selection in case a rough pass is selected. Constraint eqn. (3.30) implies that the sum of individual depths of cut are equal to total depth of cut. The above problem is an integer program which is solved using standard LINDO software. Two example are presented in the next chapter.

3.4.2 A Dynamic Programming Model

The previous model is based on two assumptions viz. equal workpiece diameter and equal tool life in all passes. The optimal number of passes are determined by counting nonzero binary variables and the optimal depth of cut corresponding to nonzero binary variable is determined q^{th} index of the binary variable. The integer programming model is, however, silent on the order in which these depths of cut should be applied. The dynamic programming (DP) approach is capable of dispensing with the two assumptions made in integer

programming model and it clearly determines the order in which the depth of cut should be applied.

The complex multipass turning problem is broken into several single pass turning problems. The total depth of cut is divided into a number of equal segments. The depth of cut for each pass is an integer multiple number of previously defined segments. The total production cost for multipass turning will be the sum of production costs for all passes. Let the multipass turning problem involve at most n_p passes which are represented as n_p stages of decision making process. Each single stage problem can be solved using single pass turning model described in section 3.2 and parametric analysis. The multipass or multistage turning problem is described in Fig. 2.3. Model 6 as stated in Chapter 2 is modified for monomial constraints and effective chip-breaking constraint as given below.

Each stage is represented by a cutting pass. The first $(n_p - 1)$ stages represent rough pass varying from 1 to $(n_p - 1)$ and the last stage n_p represents the finish pass. d_{ti} is defined as state variable representing the depth of cut remaining before stage i. In other words, d_{t1} represents the total depth of cut to be removed and $d_{t(n_p+1)}$ is equal to zero (the total depth of cut has been removed in n_p passes). The decision variables are speed, feed and depth of cut for all passes alongwith the number of passes). Minimization of production cost for multipass turning operation can be stated as multistage decision making process in the following manner:

minimize

$$C_{to} = k_{mc} + \sum_{i=1}^{i=n_p} C_{toi} \tag{3.31}$$

subject to:

(i) state transformations,

$$d_{t(i+1)} = d_{ti} - d_i \tag{3.32}$$

(ii) efficient chip-breaking region constraint,

$$f_{minlim\ i} \leq f_i \leq f_{maxlim\ i} \tag{3.33}$$

(iii) n_c monomial constraints related to each pass,

$$K_{ji} v_i^{ji} f_i^{ji} d_i ji \leq 1 \tag{3.34}$$

(iv) bounds on variables

$$v_{i\ min} \leq v_i \leq v_{i\ max} \tag{3.35}$$

$$f_{i\ min} \leq f_i \leq f_{i\ max} \tag{3.36}$$

$$d_{i\ min} \leq d_i \leq d_{i\ max} \tag{3.37}$$

where $i = 1, \ldots, n_p$ and $j = 1, \ldots, n_c$.

The total depth of cut constraint has been incorporated in multistage decision making process by the introduction of transformation equations. The production cost $C_{i+1}(d_{t(i+1)})$ at any stage $(i+1)$ is defined as the minimum production cost for stage $(i+1)$ to stage n_p and it is described as

$$C_{i+1}(d_{t(i+1)}) = \text{minimum} \ [\sum_{l=i+1}^{i=n_p} C_{toi}] \tag{3.38}$$

Further,

$$C_i(d_{ti}) = \text{minimum} \ [C_{toi} + C_{i+1}(d_{t(i+1)})] \tag{3.39}$$

subject to constraint eqns. (3.33) to (3.37).

The total depth of cut is divided into number of equal segments. The process of calculation starts from the last finish pass n_p. Cost $C_{n_p}(d_{tn_p})$ is calculated for various possible values of depth for finish pass using the single pass solution methodology described in section 3.2. The various cost coefficients A_{10} and A'_{10} are calculated. Now for the first rough pass the backward recursive relationship eqn. (3.39) is used to find $C(n_p-1)(d_{t(n_p-1)})$, and constants $A_{1(n_p-1)}$, $A'_{1(n_p-1)}$ and $A''_{1(n_p-1)}$ are calculated for various depths of cut for rough pass d_{iq}. Once these constants $A'_{1(n_p-1)}$ and $A''_{1(n_p-1)}$ have been obtained for first rough pass, these remain the same for all rough passes for the same depth of cut and cutting environment. Constant $A_{1(n_p-1)}$ can be evaluated using parametric analysis for any single stage problem formed during use of recursive relationship (3.39) for stage (n_p-2) to 1. The past practice has been to optimize a single stage problem each time it is formed [Iwata et al. 1977; Agapiou, 1992b].

3.5 Multipass Turning Solution Methodologies

The multipasss turning model described in this section with monomial constraints and effective chip-breaking region constraint alongwith bounds on variable is given as

minimize

$$C_{toi} = k_{mc} + \sum_{i=1}^{i=n_p} k_{li}(t_{pi}+t_{ai}) + \sum_{i=1}^{i=n_p} K0_{1i}t_{mi} + \sum_{i=1}^{i=n_p} K0_{2i}\frac{t_{mi}}{T_i} \tag{3.40}$$

where

$$t_{mi} = \frac{\pi D_{i-1}L}{1000 v_i f_i}$$

$$T_i = \frac{C_T^{\frac{1}{n_1}}}{v_i^{\frac{1}{n_1}} f_i^{\frac{n_2}{n_1}} d_i^{\frac{n_3}{n_1}}}$$

$$K0_{1i} = k_{li} + k_{mi}$$

$$K0_{2i} = k_{li}t_{ci} + k_{ti}$$

subject to:

(i) total depth of cut constraint,

$$\sum_{i=1}^{i=n_p} d_i - d_{t1} = 0$$

(ii) n_c number of monomial constraints,

$$K_{ji} v_i^{\alpha_{ji}} f_i^{\beta_{ji}} d_i^{\gamma_{ji}} \leq 1$$

(iii) effective chip-breaking constraint,

$$f_{minlim\ i} \leq f_i$$

$$f_i \leq f_{maxlim\ i}$$

(iv) bounds on decision variables

$$v_{i\ min} \leq v_i$$

$$v_i \leq v_{i\ max}$$

$$f_{i\ min} \leq f_i$$

$$f_i \leq f_{i\ max}$$

$$d_{i\ min} \leq d_i$$

$$d_i \leq d_{i\ max}$$

where $i = 1 \ldots n_p$ and $j = 1 \ldots n_c$.

In the previous section a dynamic programming based solution methodology for multipass turning has been presented and the optimal number of passes and depth of cut have been obtained. The present section describes various methods other than this, used for obtaining solution of multipass turning problem. An attempt has been made to compare the computational time for the methodology described in the last section with various other methods presented in this section. Various methods for minimizing multipass turning production cost are based on:

(i) Conversion of constrained multipass turning problem to unconstrained one using penalty function method and sequentially minimizing the resulting problem using Davidon, Fletcher and Powell's method (variable metric method).

(ii) Solution of constrained multipass turning model using constrained methods. Two such methods have been selected. The first method is generalized reduced gradient (GRG) method which requires a feasible starting point. The second method is sequential quadratic programming (SQP) method which can be started with any arbitrary point.

3.5.1 Sequential Unconstrained Minimization Technique (SUMT)

The constrained multipass turning operation model described in the last section is transformed into unconstrained optimization problem using penalty function given in Appendix A. The resulting problem is sequentially minimized with the help of modified parameters, used to construct the penalty function during each iteration. Davidon, Fletcher and Powell's (DFP) Method has been used for unconstrained minimization.

Davidon, Fletcher and Powell's (DFP) Method

This method is also called variable metric method. It can be thought as quasi Newton's method and also as conjugate direction method. It is a powerful general purpose optimization method making use of currently available derivatives. It starts with an initial point (which has to be a feasible point when using interior penalty method) and a positive symmetric matrix which is initially an identity matrix. The search directions are computed with the help of gradient of the function at the current point and optimal length is located using golden section method. The point thus obtained is tested for optimality. If this happens to be optimal the iterative search process is terminated otherwise matrix is updated and search is carried out using one dimensional method. The method converges quadratically because it is a conjugate gradient method.

Sometimes it is not possible to find an initial feasible point easily. A separate optimization problem is then formulated to obtain the initial feasible solution. This initial solution is taken as a starting point and penalty function is formed with the help of suitable parameters. The penalty function is now minimized using DFP. The parameters of penalty function are changed sequentially and new penalty function is formed till no significant change in objective function occurs. The complete methodology is called Sequential Unconstrained Minimization Technique (SUMT) with DFP. This methodology has been implemented in FORTRAN 77 and is discussed in Appendix A.

3.5.2 Constrained Programming Methods

Constrained programming methods or primal methods are those methods where a constrained optimization problem is solved directly. It has been reported that one such constrained method called generalized reduced gradient (GRG) requires less execution time than SUMT using DFP for single pass milling operation with non-linear objective function and monomial constraints [Eskcioglu et al., 1992]. However this method also requires an initial feasible point. Another constrained method which may be started from an arbitrary point is called sequential quadratic programming method (SQP). In this method the objective function as well as constraints are linearized about a selected point and linearized

objective function is made quadratic by adding a quadratic step length. Thus the resulting quadratic programming problems are sequentially solved.

Generalized Reduced Gradient Method

The method hereafter abbreviated as GRG is due to Abadie and Carpentier [1969]. This method is a natural extension of Wolfe's reduced gradient method. Wolfe solves problems with linear constraints and non-linear objective function. GRG as an extension to this method is concerned with non-linear equality constraints. The non-linear inequality constraints are converted to equality constraints by adding slack variables. The complete code has been prepared and implemented in FORTRAN 77. The general idea about the GRG algorithm and necessary details for its implementation are given in Appendix B.

Sequential Quadratic Programming Method

In this method the objective function as well as constraints are linearized about some point. The resulting linear objective function is made quadratic by addition of quadratic step size. The quadratic step size is added to eliminate the need for approximation of linear move limits. Thus the resulting quadratic programming problem has been solved using augmented Lagrange multiplier method. The solution of one quadratic programming problem is used to generate next quadratic programming problem and therefore the method is called sequential quadratic programming method (SQP). More details have been provided in Appendix C.

Sl. No.	Description of the Method	No. of Passes Known/Decision Variable	Simultaneous Multipass or Single Pass Solutions	Remarks
1	Continuous variable optimization	Known	Simultaneous	Single pass solution methods may also be used for optimization predecided depth of cut distribution
2	Continuous variable optimization	Continuous decision variable	Simultaneous	The resulting real optimal number of passes is rounded off to nearest integer depending on minimum production cost
3	Reformulate the problem for assumed maximum number of passes with additional binary variables and a few constraints and use continuous variable optimization	The optimal number of passes are obtained by counting nonzero binary variables in solution	Simultaneous	--------
4	Reformulate the problem on the basis of production cost vs. depth of cut trends for assumed maximum number of passes with a few binary variables and constraints and solve resulting integer programming problem	The optimal number of passes are obtained by counting nonzero binary variables in solution	Single pass solutions to obtain production cost trends	--------
5	Multistage decision making or dynamic programming	Discrete variable	Single pass solutions	Results in discrete depth of cut distribution. Refinement in depth of cut distribution requires a large number of single pass solutions
6	Mixed variable optimization	Discrete variable	Simultaneous	Not found in reviewed literature

Table 3.1: Various methodologies for obtaining optimal machining conditions for multipass turning operations.

Chapter 4

Numerical Results for Multipass Straight Turning

4.1 Introduction

In section 3.2 of previous chapter a methodology to minimize the production cost for multipass turning problem has been presented. It is based on the determination of optimal tool life corresponding to minimum production cost under monomial constraints using one dimensional golden section search method. It has been observed through eqn. (3.3) that the minimum production cost at a given tool life for a given depth of cut is obtained by using maximum allowable feed under the constraints for single pass turning. The present work aims at solving multipass turning problem by using single pass solutions. This means finding the optimal number of passes and depth of cut distribution corresponding to all passes. The speeds and feeds for all passes are also to be determined using single pass solutions. Two models based on integer programming and dynamic programming have been presented in sections 3.4.1 and 3.4.2, respectively. These models help in synthesis of results of multipass turning problem through single pass solutions. The integer programming model is based on constant initial workpiece diameter in all passes, whereas the dynamic programming model is able to treat varying workpiece diameter. The minimum production cost corresponding to a series of feasible depths of cut for rough as well as finish passes has to be obtained for both the models using single pass solution methodology described in section 3.2.

Integer programming model of multipass turning problem is formulated on the basis of one final finish and assumed maximum number of rough passes. Series of possible depths of cut for finish as well as rough passes are made. The minimum production cost for each depth of cut in series is obtained using the single pass solution methodology (section 3.2) and a binary variable is associated with each depth. The multipass cost minimization model is transformed to linear integer programming model where the cost function is obtained using minimum production costs and variables and constraints are constructed such that only one finish pass and a number of rough passes are selected in order to cut the total depth.

The optimal number of passes are obtained counting non-zero variables in the solution and associated depths are declared as optimal. The speeds and feeds corresponding to optimal depths are already available from single pass solutions.

The common practice has been to use small depth of cut in finish pass and to remove the remaining depth of cut in a number of rough passes of equal depth yielding minimum production cost. The multipass turning problem for all feasible total number of passes are formulated. The minimum production cost corresponding to each such formulation is obtained and the lowest among these formulations is selected as optimal production cost and corresponding total number of passes are declared as optimal number of passes. Shin and Joo [1992] have suggested that the small value for finish pass depth should be kept at the least allowable value of depth of cut to get the least value of production cost in finish pass.

In this chapter five examples have been considered for the purpose of illustration. The first two examples show the use of integer programming model and compare the results obtained using integer programming model and commonly practised strategy as suggested by Shin and Joo [1992]. The second example has also been solved using dynamic programming to compare with the results obtained previously using integer programming and commonly practised strategy. The same example is called example 3 when it is solved on the basis of varying workpiece diameter using dynamic programming model. These three examples have been solved at a given tool life in all passes in order to compare the results of three models viz. Shin and Joo's model, integer programming model and dynamic programming model. The dynamic programming model as developed in the present work is able to handle varying workpiece diameter and non-uniform tool life in all passes. Example 4 has been taken as a general one and it has been found that, in general, the optimal tool life for finish pass is different from that obtained for rough passes. The fifth example has been taken to compare computer time for three other nonlinear methods viz. SUMT with DFP, GRG and SQP with dynamic programming model developed in the present work.

4.2 Integer Programming Model

This model is based on same workpiece diameter in rough as well as finish passes. Further, it has been assumed that the optimal tool life for all passes are equal. The production costs for various depths of cut has been obtained on this assumption. The integer programming model works in two phases as described in section 3.4.1. In the first phase minimum production cost for a series of feasible depths of cut in finish as well as rough passes have been obtained. An integer programming problem equivalent to multipass turning is created in the second phase and this is solved using LINDO software. The following two examples have been solved using integer programming model and the resulting depth of cut distribution

yields minimum production cost in majority of cases. The integer programming formulati has been solved using LINDO software which sometimes terminate to non-optimal a non-feasible solutions. In such cases the integer programming formulation is not able reach optimal production cost whereas dynamic programming formulation yields minimu production cost.

In the proposed model, it has been assumed that the workpiece diameter is same in the passes and both rough and finish pass tools have the same tool life. These assumpti have been made to make problem formulation easy. The proposed model gives the value optimum number of passes using branch and bound method. It has been found that t software LINDO sometimes fails to terminate at the optimal solution. Moreover, the dep of cut values obtained when applied in different order will actually result in different co when seen in conjunction with the varying diameter model. The model is silent about t order in which these optimal depth of cut should be applied.

4.2.1 Example 1

The multipass cylindrical turning example given by Shin and Joo [1992] is considered this example. The data as used by Shin and Joo [1992] are given below.

- Blank diameter = 50 mm.
- Length of the workpiece = 300 mm.
- Total depth of cut to be removed = 6 mm.
- Allowable cutting speed range, $v_{i\ min} = 5$ m/min; $v_{i\ max} = 500$ m/min.
- Allowable cutting feed range, $f_{i\ min} = 0.1$ mm/rev; $f_{i\ max} = 0.9$ mm/rev.
- Allowable cutting depth range, $d_{i\ min} = 1.0$ mm; $d_{i\ max} = 3.0$ mm.
- Constant for tool life equation, $C_{Ti}^5 = 6 \times 10^{11}$.
- Exponents for tool life equation, $\alpha_{0i} = 5.0$; $\beta_{0i} = 1.75$; $\gamma_{0i} = 0.75$ or $n_{1i} = 0$ $n_{2i} = 0.35$; $n_{3i} = 0.15$.
- Allowable cutting tool life range, $T_{i\ min} = 25$ min; $T_{i\ max} = 45$ min.
- Constant for cutting force equation, $K'_{1i} = 1.059$
- Exponents for cutting force equation, $\alpha_{1i} = 0$, $\beta_{1i} = 0.75$; $\gamma_{1i} = 0.95$.
- Maximum allowable cutting force, $F_{i\ max} = 1.962$ kN.
- Cutting power equation constant, $K'_{2i} = 0.02076$.

- Exponents for cutting power equation, $\alpha_{2i} = 1.0$, $\beta_{2i} = 0.75$; $\gamma_{2i} = 0.95$.

- Maximum allowable cutting power, $P_{i\ max} = 5$ kW.

- Constant for surface finish equation, $K'_{3i} = 0.125/r$; nose radius $r = 1.2$ mm.

- Exponents for surface finish equation, $\alpha_{3i} = 0$; $\beta_{3i} = 2.0$. $\gamma_{3i} = 0$.

- Maximum allowable surface roughness for finish and rough passes $R_{0\ max} = 10\ \mu$m; $R_{i\ max} = 100\ \mu$m.

- Constants for tool approach and detraction time, $h_{1i} = 7 \times 10^{-4}$; $h_{2i} = 0.3$ min.

- Time components, $t_{p1} = 0.75$ min/piece; $t_{ci} = 1.5$ min/cutting edge.

- Cost components, $k_{li} = \$\ 0.5$/min; $k_{ti} = \$\ 2.5$/cutting edge.

For the above data eqn. (3.4) gives $A0_{1i} = 0.249$, assuming $T_i = 25$ min (test point), $k_{li}t_{ai} = 0.255$ and $k_{l1}t_{p1} = 0.375$ has to be added to the final solution once only.

For total depth of cut values upto 10.0 mm, the maximum number of rough passes are assumed to be 4. This is because the minimum depth of cut which can be removed in finish pass is 1 mm and therefore minimum number of rough passes required for Shin and Joo's method are 3. Therefore the value of assumed maximum number passes has been taken to be more than 3. Thus, integer programming problem can be formulated even for a larger number. The number of integer variables is more when a large value for maximum number of passes is selected. The result of integer programming problem remains unaffected with the choice of this number and therefore the smaller numbers should be preferred.

The value of m_i for $i = 0, 1 \ldots . n$ is taken as 20 for the generation of depth of cut series. Therefore,

$$
\begin{aligned}
d_{i0} &= 1.0 \text{ mm} \\
d_{i1} &= 1.0 + (3.0 - 2.0)\tfrac{1}{20} = 1.1 \text{ mm} \\
d_{i2} &= 1.2 \text{ mm} \\
d_{i3} &= 1.3 \text{ mm} \\
&------ \\
&------ \\
d_{i20} &= 3.0 \text{ mm.}
\end{aligned}
$$

The minimization of total cost C_{to} can be stated as

minimize

$$C_{to} = \sum_{i=0}^{i=n} C_{toi} \tag{4.1}$$

where the value of C_{toi} for finish or rough pass as given by eqn. (3.3) is

$$C_{toi} = A0_{1i} f_{iq}^{-(1-n_{2i})} d_{iq}^{n_{3i}} + 0.255 \tag{4.2}$$

The cost for finish pass and rough pass are minimized separately under the constraints for that pass using the methodology given in section 3.2. The two phases of the problem can now be defined.

Phase 1

The cost minimization for finish pass may be stated as

minimize

$$C_{to0} = 0.249 f_{0q}^{-0.65} d_{0q}^{0.15} + 0.255 \tag{4.3}$$

subject to the following constraints:

(i) Cutting force constraint

$$0.54 f_{0q}^{0.75} d_{0q}^{0.95} \leq 1 \tag{4.4}$$

(ii) Cutting power constraint

$$0.4152 \times 10^{-2} v_{0q} f_{0q}^{0.75} d_{0q}^{0.95} \leq 1 \tag{4.5}$$

(iii) Surface finish constraint

$$f_{0q} \leq 0.31 \tag{4.6}$$

(iv) Bounds on speed, feed and depth of cut

$$5 \leq v_{0q} \leq 500 \tag{4.7}$$

$$0.1 \leq f_{0q} \leq 0.9 \tag{4.8}$$

Combining eqns. (4.6) and (4.8) the constraints for feed and depth of cut assume the form

$$0.1 \leq f_{0j} \leq 0.31 \tag{4.9}$$

$$1.0 \leq d_{0q} \leq 3.0 \tag{4.10}$$

The maximum permissible feed f_{0q}^{*} under the constraints to obtain the minimum production cost at the test point (25 min) is obtained using steps 1 to 3 described in section 3.2. Thereafter the minimum production cost and the speed corresponding to minimum production cost are obtained using steps 4 and 5. In addition to this, the values for variable production cost per unit diameter $C_{tovar\ 0q}^{*'}$ and variable production cost per unit diameter per unit length $C_{tovar\ 0q}^{*''}$ are also calculated for a given value of depth of cut d_{0q}. These values for all feasible depths of cut for finish pass are listed in Table 4.1.

All constraints are same for rough passes except the value of $R_{i\ max}$ and the surface finish constraint takes the form

$$f_{iq} \leq 0.979 \tag{4.11}$$

The constraint of surface finish therefore does not make any change on the bounds of feed and the production cost minimization problem for any rough pass is stated as

minimize

$$C_{toi} = 0.249 f_{iq}^{-0.65} d_{iq}^{0.15} + 0.255 \tag{4.12}$$

subject to the following constraints:

(i) Cutting force constraint

$$0.54 f_{iq}^{0.75} d_{iq}^{0.95} \leq 1 \tag{4.13}$$

(ii) Cutting power constraint

$$0.4152 \times 10^{-2} v_{iq} f_{iq}^{0.75} d_{iq}^{0.95} \leq 1 \tag{4.14}$$

(iii) Bounds on speed, feed and depth of cut

$$5 \leq v_{iq} \leq 500 \tag{4.15}$$

$$0.1 \leq f_{iq} \leq 0.9 \tag{4.16}$$

$$1.0 \leq d_{iq} \leq 3.0 \tag{4.17}$$

The maximum permissible feed under the constraint for rough pass f_{iq}^* is obtained in the manner similar to that for finish pass. This feed value minimizes the minimum production cost $C_{to\ iq}^*$. In addition to the minimum production cost, the minimum variable production cost per unit diameter $C_{tovar\ iq}^{*'}$, the minimum variable production cost per unit diameter per unit length $C_{tovar\ iq}^{*''}$, and the cutting speed v_{iq}^* are obtained as for finish pass for each value of depth of cut. These values for each value of depth of cut d_{iq} are listed in Table 4.2.

Phase 2

In this phase the above problem is formulated as a linear integer programme as described in phase 2 of section 3.4.1 and is solved using LINDO software. The results obtained are given in Table 4.3. The results for the same problem have also been obtained using Shin and Joo's method. These are also given in Table 4.3. The integer programming method indicates that one rough pass and one finish pass are required. The depth of cut for both the passes is 3.0 mm and the minimum production cost is \$ 1.94/piece. The method of Shin and Joo [1992] requires two rough passes of 2.5 mm and one finish pass of 1.0 mm depth and the corresponding minimum production cost is \$ 2.385/piece. Therefore the integer programming method determines lower production cost than that obtained by Shin and Joo. The total number of passes required in the proposed integer programming method is also lower or equal to that found using Shin and Joo's method.

Effect of Varying Total Depth of Cut on Minimum Cost

The above problem has been solved for various values of total depth of cut ranging from 6.0 to 11.0 mm. The results of the proposed integer programming method and the method of Shin and Joo are presented in Table 4.3 for comparison purposes. For all depths of cut, the proposed model results in lower production cost than the method of Shin and Joo. The optimum number of passes in the proposed method are also either lower or equal to that found by Shin and Joo.

4.2.2 Example 2

In example 1, the depth of cut for both finish and rough passes has been taken in the range of 1.0 to 3.0 mm. However, from the practical point of view this depth of cut range for the finish pass seems to be on the higher side and new range is taken as 0.3 to 1.2 mm [Hinduja et al., 1985]. Thus, $d_{0min} = 0.3$ mm and $d_{0max} = 1.2$ mm. The depth of cut range for the rough pass and other data are the same. The two phases for the solution are described below

Phase 1

For $d_{t1} = 10.0$ mm, the maximum number of passes is again taken as 4. The value of m_0 is taken as 8 for generating the depth of cut series for finish pass, i.e.,

$$d_{00} = 0.3 \text{ mm}$$
$$d_{01} = 0.4 \text{ mm}$$
$$\text{------}$$
$$\text{------}$$
$$d_{09} = 1.2 \text{ mm}.$$

The minimum production cost for finish pass, $C^*_{to\ 0q}$, is found along with the optimum cutting speed v^*_{0q}, feed f^*_{0q} for d_{0q}; $q = 0, 1.\dots m_0$. In addition to this the values for $C^{*'}_{tovar\ 0q}$ and $C^{*''}_{tovar\ 0q}$ are also calculated corresponding to each d_{0q}. These are listed in Table 4.4. The values of $m_i = 1 \dots n$ are taken as 20 as in example 1 and the same d_{iq} series is generated. Further, the values $C_{to\ iq}$, v^*_{iq} and f^*_{iq} have not been calculated for this series. They are already available in Table 4.2.

Phase 2

The above problem is formulated as an integer programming problem and is solved using LINDO software as for example 1. The results for this example using the integer programming method and Shin and Joo's Method are presented in Table 4.5.

For d_{t1} = 6.0 to 7.0 mm, the proposed integer programming model gives reduced number of passes and reduced minimum production cost than those obtained using Shin and Joo's method (or commonly practised strategy). For d_{t1} = 7.5 mm the proposed model does not achieve minimum production cost probably because Lindo software sometimes terminates at a non-optimal and/or non-feasible solution. It will be noted that the solution is feasible but it may be non-optimal (see discussions of dynamic programming model). The solution of Shin and Joo's method is available in feasible discrete space set.

The proposed model gives slightly higher production cost of \$ 2.876/per piece as compared to \$ 2.766/piece obtained using the other method.

The proposed model gives slightly higher minimum production cost of \$2.946/piece at d_{t1} = 8.0 mm. This time the feasible discrete space in the proposed model does not contain the minimum cost point obtained by the other method. The proposed model successfully achieves either equal or reduced minimum production cost of \$ 3.006/piece and \$ 3.020/piece, respectively for d_{t1} = 8.5 mm and d_{t1} = 9.0 mm. Further, at d_{t1} = 9.5 mm or 10.0 mm the proposed model gives reduced number of passes and reduced minimum production cost (in discrete feasible space) than those obtained using the other method. Thus, out of 9 cases considered above, the proposed model clearly gives minimum production cost at maximum of finishing depth range, i.e., 1.2 mm for 5 cases by reducing number of passes. For d_{t1} = 7.5 to 9.5 mm, the proposed model gives almost equal or reduced minimum production cost for two cases and failed in remaining two cases. These two cases will be examined in detail using dynamic programming model.

4.3 Dynamic Programming Model

The integer programming model is based on constant initial workpiece diameter whereas the dynamic programming model described in section 3.4.2 is able to accommodate varying workpiece diameters at each pass. It can also be adapted to constant initial workpiece diameter. The solution procedure for dynamic programming model is given below.

4.3.1 Example 3

In this example the data is the same as in example 2. It has been assumed that the tool life for both rough as well as finish pass is the same and is equal to 25 min. This assumption has been made in order to compare the results obtained using dynamic programming model with those obtained previously using integer programming model or Shin and Joo's method where tool life in all passes has been assumed to be the same.

Solution Procedure

The initial workpiece diameter is 50 mm. If the total depth of cut to be removed is 6 mm, the final workpiece diameter will be $50-2\times6=38$ mm. Thus 6 mm depth is divided into 60 divisions of 0.1 mm (the number of divisions depend upon the desired accuracy). Similarly, the depth of cut ranges for finish as well as rough passes are converted into number of divisions as 3 to 12 and 10 to 30, respectively.

Let a matrix element $cost_{kq}$ be defined as minimum production cost for machining total k divisions when the depth of cut corresponding to q^{th} division is machined in rough pass. The value of q has been defined in section 4.2.1. The minimum cost for machining k divisions, $cost_{min\ k}$ is given as

$$cost_{min\ 1} = \text{minimum } cost_{kq} \text{ for all q} \tag{4.18}$$

$cost_{min\ 1}$ and $cost_{min\ 2}$ are given a very high value of 9999999 because finish pass of 0.1 and 0.2 mm depths of cut are not feasible.
$cost_{min\ 3}$ to $cost_{min\ 12}$ are calculated as

$$cost_{min\ k} = (.02k + D_f)C^{*'}_{tovar\ 0(k-3)} + 0.255 \tag{4.19}$$

where the value of $C^{*'}_{tovar\ 0(k-3)}$ is obtained from Table 4.1. Using the relationship

$$cost_{kq} = (0.02k + D_f)C^{*'}_{tovar\ iq} + cost_{min}(index) + 0.255 \tag{4.20}$$

$cost_{kq}$ for k = 13 to 60 and q = 0 to 20 are obtained provided $q = (k - (3 + 10))$ is non-negative. Here $C^{*'}_{tovar\ iq}$ is obtained from Table 4.2, $index$ is given as

$$index = k - (0.3 + q) \tag{4.21}$$

and $cost_{kq}$ = 9999999 if term $[k - (3 + 10)]$ is negative. $cost_{min\ k}$ is then obtained and the value of q for which $cost_{min\ k}$ corresponds is called key_k. This key is used for backtracking the depth of cut distribution. $cost_{min\ 60}$ is found and key_{60} gives the value of depth of cut for the first rough pass and 60 - key_{60} gives the remaining depth of cut and $key_{60-key_{60}}$ will give the depth of cut for the second pass and so on.

Observations

The values for the minimum production cost for various depths of cut are given along with the depth of cut distribution for finish pass as well as rough pass in Table 4.5. The dynamic programming model has been adapted for constant diameter model using D_0 in place of $(0.02k + D_f)$ in eqns (4.19) and (4.20). The results for various depths of cut have been presented in Table 4.6 and it is clear that for all depths of cut ranging from 6 to 10 mm the minimum production cost obtained using dynamic programming model is lower than that obtained using Shin and joo's method or integer programming model.

It is important to note that for a depth of cut of 7.5 mm the integer programming model is not able to achieve lower minimum production cost than \$ 2.872/piece as obtained by Shin and Joo's method (see Table 4.5) but the dynamic programming model gives lower minimum production cost of \$ 2.843/piece (see Table 4.6). Similarly, for a depth of cut of 8 mm also when integer programming model is not able to achieve the minimum production cost, the dynamic programming model yields minimum production cost. The most probable reason for these kind of incidents is that LINDO software used in the second phase of integer programming model sometimes gives non-optimal results. It is also clear from the results given in Table 4.6 that the minimum production cost with varying workpiece diameter gives lower value than that obtained using initial workpiece diameter. Thus it is evident that dynamic programming model gives more accurate results than integer programming model. Further, integer programming model does not give any information about the order in which the depths of cut should be applied but the dynamic programming model gives the order in which these optimal depths of cut should be applied.

4.3.2 Example 4

All the three examples presented earlier were minimizing the production cost at tool life of 25 min which remain the same for both finish and rough passes. The assumption of equal tool life in all passes is, in general, not true but it has been used in these three examples to compare the results of dynamic programming model, integer programming model and Shin and Joo's method. Therefore in this example the tool life and constraint data has been taken partially from Ermer and Kromodihardjo [1981] and the cost and time coefficients are more or less the same as in example 1. The example data are as follows:

- Blank diameter = 50 mm.
- Length of the workpiece = 300 mm.
- Total depth of cut to be removed = 6 mm.
- Allowable cutting speed range, $v_{i\ min}$ = 5 m/min; $v_{i\ max}$ = 500 m/min.
- Allowable cutting feed range, $f_{i\ min}$ = 0.1 mm/rev; $f_{i\ max}$ = 0.9 mm/rev.
- Allowable cutting depth range for finish pass, $d_{0\ min}$ = 0.3 mm; $d_{0\ max}$ = 1.2 mm.
- Allowable cutting depth range for rough passes, $d_{i\ min}$ = 1.0 mm; $d_{i\ max}$ = 3.0 mm.
- Constant for tool life equation, $K_{0i} = 1.40673 \times 10^9$.
- Exponents for tool life equation, $\alpha_{0i} = 4.0$; $\beta_{0i} = 1.16$; $\gamma_{0i} = 0.15$.

- Allowable cutting tool life range, $T_{i\ min} = 5$ min; $T_{i\ max} = 60$ min.
- Cutting power equation constant, $K'_{1i} = 0.499$.
- Exponents for cutting power equation, $\alpha_{1i} = 0.95$, $\beta_{1i} = 0.78$; $\gamma_{1i} = 0.75$.
- Maximum allowable cutting power, $P_{i\ max} = 20$ kW.
- Constant for surface finish equation, $K'_{2i} = 0.125/R_{i\ max}$ nose radius $r = 1.2$ mm.
- Exponents for surface finish equation, $\alpha_{2i} = 1.52$ $\beta_{2i} = 1.004$ $\gamma_{2i} = 0.25$
- Maximum allowable surface roughness for finish and rough passes $R_{0\ max} = 5\ \mu$m; $R_{i\ max} = 25\ \mu$m.
- Constants for tool approach and detraction time, $h_{1i} = 7 \times 10^{-4}$; $h_{2i} = 0.3$ min.
- Time components, $t_{p1} = 0.75$ min/piece; $t_{ci} = 1.5$ min/cutting edge.
- Cost components, $k_{li} = \$\ 0.5$/min; $k_{ti} = \$\ 2.5$/cutting edge.

In this example the tool life, cutting power and surface finish equations have been taken from Ermer and Krmodihardjo [1981].

Observations

The minimum production cost for different depths of cut ranging from 0.3 to 1.2 mm are listed in Table 4.7. The optimal tool life corresponding to various depths of cut changes from 15.78 to 38.85 min in finish pass depending upon the depth of cut used.

The minimum production cost for various depths of cut ranging from 1.0 to 3.0 mm in rough pass are given in Table 4.8. The optimal tool life corresponding to various depths of cut are the same (59.97 min). It may be noted that this value is significantly different than that obtained in the finish pass (15.78 to 38.85 min). Therefore the assumption of equal tool life in all passes made to simplify the solution of multipass turning problem will lead to incorrect results. The minimum production cost in this case is \$ 4.4457/piece based on variable workpiece diameter and without any restriction on tool life in any pass. The optimum machining conditions are as following:

$$\begin{aligned}
d_0^{opt} &= 1.2 \text{ mm}; \; v_0^{opt} = 143.9 \text{ m/min}; \; f_0^{opt} = 0.853 \text{ mm/rev}; \\
d_1^{opt} &= 2.3 \text{ mm}; \; v_1^{opt} = 83.93 \text{ m/min}; \; f_1^{opt} = 0.231 \text{ mm/rev} \\
d_2^{opt} &= 2.5 \text{ mm}; \; v_2^{opt} = 83.87 \text{ m/min}; \; f_2^{opt} = 0.214 \text{ mm/rev}
\end{aligned}$$

4.4 Comparison of Various Nonlinear Techniques

The various nonlinear programming techniques for minimization of production cost have been described in section 3.5. These are named as sequential unconstrained minimization technique (SUMT) with Davidon, Fletcher and Powell's (DFP) method, generalized reduced gradient (GRG) method and sequential quadratic programming (SQP) method.

4.4.1 Example 5

The objective function chosen by Chua et al. [1993] is minimum production time and only rough turning is considered. The production cost function results in production time function if overhead cost is taken as unity and machining as well as tool edge cost are taken to be zero. Further, the surface finish requirement is slackened or a high value of R_{max} is selected for both rough and finish passes. The following data are used:

The data used for machining of Röchling T4(C 0.45%, Si 0.25%, Mn 0.70%) with SANDVIK 42 TiN-coated carbide insert (SNUN 120408) by Chua et al. [1993] are given below.

- Blank diameter = 200 mm.
- Length of the workpiece = 400 mm.
- Total depth of cut to be removed = 4 mm.
- Allowable cutting speed range, $v_{i\ min}$ = 150 m/min; $v_{i\ max}$ = 212 m/min.
- Allowable cutting feed range, $f_{i\ min}$ = 0.2 mm/rev; $f_{i\ max}$ = 0.35 mm/rev.
- Allowable cutting depth range, $d_{i\ min}$ = 1.0 mm; $d_{i\ max}$ = 2.0 mm.
- Constant for tool life equation, $K_{0i} = 8.81 \times 10^7$.
- Exponents for tool life equation, $\alpha_{0i} = 2.894$; $\beta_{0i} = 0.493$; $\gamma_{0i} = 0$.
- Allowable cutting tool life range, $T_{i\ min}$ = 27.373755 min; $T_{i\ max}$ = 98.165 min.
- Constant for cutting force equation, $K'_{1i} = 10.1$
- Exponents for cutting force equation, $\alpha_{1i} = -.0.171$, $\beta_{1i} = 0.641$; $\gamma_{1i} = 0.852$.
- Maximum allowable cutting force, $F_{i\ max}$ = 12 kN.
- Cutting power equation constant, $K'_{2i} = 0.179$.
- Exponents for cutting power equation, $\alpha_{2i} = 0.814$, $\beta_{2i} = 0.634$; $\gamma_{2i} = 0.845$.
- Maximum allowable cutting power, $P_{i\ max}$ = 3 kW.

- Constant for surface finish equation, $K'_{3i} = 0.125/r$; nose radius $r = 1.2$ mm.
- Exponents for surface finish equation, $\alpha_{3i} = 0$; $\beta_{3i} = 2.0$. $\gamma_{3i} = 0$.
- Maximum allowable surface roughness for finish and rough passes, $R_{0\ max} = 2500\ \mu$m $R_{i\ max} = 2500\ \mu$m.
- Constants for tool approach and detraction time, $h_{1i} = 0.0$; $h_{2i} = 0.0$ min.
- Time components, $t_{p1} = 1.0$ min/piece; $t_{ci} = 1.0$ min/cutting edge.
- Cost components, $k_{li} = \$\ 1.0$/min; $k_{ti} = \$\ 0.0$/cutting edge.

(It may be noted that this is a production time minimization problem because K_{li} and t_{ci} are unity and k_{ti} are zero).

4.4.2 Observations

All the above mentioned techniques require the number of passes to be known for the formulation of optimization problem. $n_{min} = 2$ $n_{max} = 4$ for the above problem. Therefore the multipass turning model are to be formulated and solved for 2, 3 and 4 number of passes. Otherwise three problems of similar nature have to be solved as suggested by Chua et al. [1993]. In the first problem, the number of passes are treated as continuous variable and an optimum continuous approximation of the number of passes has been obtained as 2.58. The next two problems are formulated for two nearest integer to the continuous optimum number of passes, i.e., 2 and 3. The first two methods require feasible starting point for which a separate optimization problem is to be solved. SQP may be started from an arbitrary point and dynamic programming alongwith single pass solution methodology does not require any starting point. SUMT with DFP method is dependent on optimization parameters. Similarly, GRG is also dependent on optimization parameters and convergence of the method depends upon the initial partition of basic and non-basic variables. On the otherhand, no such parameter are required for SQP method. The dynamic programming method neither requires any optimization parameter nor it requires any knowledge of number of passes. Rather the dynamic programming method determines the number of passes uniquely. The optimum distribution of depth of cut is also successfully determined. Therefore this method should be preferred over the other three because of three reasons: (i) the number of passes are determined uniquely, (ii) no prior knowledge of feasible starting point is required and (iii) no optimization parameter is required. The computation time which is the sum of user time and system time have been compared for the three non-linear techniques alongwith the dynamic programming model using single pass solution methodology. It has been found that SQP requires the least time, that is approximately $3 \times 0.1 = 0.3$ seconds. This methodology may not be available to all manufactures. The dynamic programming methodology

requires 0.9 seconds but is easy to implement. GRG method ranks the third and it requires $3 \times 0.4 = 1.2$ seconds and SUMT with DFP requires $3 \times 0.8 = 2.4$ seconds. It may be noted that the last two methods GRG and SUMT with DFP require initial feasible solution also. The dynamic programming method should, therefore, be preferred over other methods.

S.No.	d_{0q} mm	v^*_{0q} m/min	f^*_{0q} mm/rev	$C^{*'}_{tovar\ 0q}$ \$/piece 10^{-3}	$C^{*''}_{tovar\ 0q}$ \$/piece 10^{-5}	C^*_{to0q} \$/piece
0	1.0	179.5	0.31	10.66	3.553	0.788
1	1.1	177.0	0.31	10.82	3.607	0.796
2	1.2	174.7	0.31	10.96	3.653	0.803
3	1.3	172.6	0.31	11.08	3.693	0.809
4	1.4	170.7	0.31	11.22	3.740	0.816
5	1.5	168.9	0.31	11.34	3.780	0.822
6	1.6	167.3	0.31	11.44	3.813	0.827
7	1.7	165.8	0.31	11.54	3.847	0.832
8	1.8	164.4	0.31	11.64	3.880	0.837
9	1.9	163.0	0.31	11.74	3.913	0.842
10	2.0	161.8	0.31	11.84	3.947	0.847
11	2.1	160.6	0.31	12.00	4.000	0.855
12	2.2	159.5	0.31	12.00	4.000	0.855
13	2.3	158.4	0.31	12.08	4.027	0.859
14	2.4	157.4	0.31	12.16	4.053	0.863
15	2.5	156.4	0.31	12.24	4.080	0.867
16	2.6	155.5	0.31	12.30	4.100	0.870
17	2.7	154.7	0.31	12.38	4.127	0.874
18	2.8	153.8	0.31	12.44	4.147	0.877
19	2.9	153.0	0.31	12.50	4.167	0.880
20	3.0	152.2	0.31	12.58	4.193	0.884

Table 4.1: Optimal cutting conditions and costs when the total stock is removed in a single finish pass at $T_0 = 25$ min (Example 1).

S.No.	d_{iq} mm	v^*_{iq} m/min	f^*_{iq} mm/rev	$C^{*'}_{tovar\ iq}$ \$ 10^{-3}/piece	$C^{*''}_{tovar\ iq}$ \$ 10^{-5}/piece	C^*_{toiq} \$/piece
0	1.0	123.6	0.9	5.34	1.780	0.522
1	1.1	121.8	0.9	5.40	1.800	0.525
2	1.2	120.3	0.9	5.48	1.827	0.529
3	1.3	118.8	0.9	5.54	1.847	0.532
4	1.4	117.5	0.9	5.60	1.867	0.535
5	1.5	116.3	0.9	5.67	1.887	0.538
6	1.6	115.2	0.9	5.72	1.901	0.541
7	1.7	114.1	0.9	5.78	1.927	0.544
8	1.8	113.2	0.9	5.82	1.940	0.546
9	1.9	112.3	0.9	5.88	1.960	0.549
10	2.0	111.4	0.9	5.92	1.970	0.551
11	2.1	111.1	0.889	6.00	2.000	0.555
12	2.2	112.6	0.838	6.28	2.093	0.569
13	2.3	114.1	0.792	6.56	2.187	0.583
14	2.4	115.5	0.750	6.84	2.280	0.597
15	2.5	116.9	0.712	7.12	2.370	0.611
16	2.6	118.3	0.678	7.40	2.407	0.625
17	2.7	119.6	0.646	7.68	2.560	0.639
18	2.8	120.9	0.617	7.96	2.650	0.653
19	2.9	122.2	0.590	8.24	2.747	0.667
20	3.0	123.4	0.565	8.52	2.840	0.681

Table 4.2: Optimal cutting conditions and costs when the total stock is removed in a single rough pass at $T_i = 25$ min (Example 1).

		Proposed model						Shin and Joo's model			
S.No	d_{t1} mm	d_0^{opt} mm	d_1^{opt} mm	d_2^{opt} mm	d_3^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece	d_0^{opt} mm	d_i^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece
1	6.0	3.0	3.0			1	1.940	1.0	2.50	2	2.385
2	8.0	3.0	2.1	2.9		2	2.481	1.0	2.33	3	2.927
3	8.5	3.0	2.5	3.0		2	2.551	1.0	2.50	3	2.996
4	9.0	3.0	3.0	3.0		2	2.611	1.0	2.67	3	3.068
5	9.5	2.9	1.9	2.8	1.9	3	3.005	1.0	2.83	3	3.134
6	10.0	3.0	2.1	2.8	2.1	3	3.022	1.0	3.00	3	3.206
7	10.5	2.7	2.0	2.9	2.9	3	3.134	1.0	2.50	4	3.539
8	11.0	3.0	2.9	2.1	3.0	3	3.162	1.0	2.50	4	3.607

Table 4.3: Comparison of results obtained for the proposed integer programming model and Shin and Joo's model at $T_i = 25$ min (Example 1).

S.No.	d_{0q} mm	v_{0q}^* m/min	f_{0q}^* mm/rev	$C_{tovar\ 0q}^{*'}$ \$/piece	$C_{tovar\ 0q}^{*''}$ \$/piece	$C_{tovar\ 0q}^{*}$ \$/piece
0	0.3	215.0	0.31	8.9	2.967	0.700
1	0.4	205.9	0.31	9.3	3.100	0.720
2	0.5	199.2	0.31	9.6	3.200	0.735
3	0.6	193.8	0.31	9.88	3.293	0.749
4	0.7	189.4	0.31	10.10	3.367	0.760
5	0.8	185.6	0.31	10.32	3.440	0.771
6	0.9	182.4	0.31	10.50	3.500	0.780
7	1.0	179.5	0.31	10.66	3.553	0.788
8	1.1	177.0	0.31	10.82	3.607	0.796
9	1.2	174.7	0.31	10.96	3.653	0.803

Table 4.4: Optimal cutting conditions and costs for modified finish pass range at $T_0 = 25$ min (Example 2).

		Proposed model						Shin and Joo's model			
S.No	d_{t1} mm	d_0^{opt} mm	d_1^{opt} mm	d_2^{opt} mm	d_3^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece	d_0^{opt} mm	d_i^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece
1	6.0	1.2	2.7	2.1		2	2.372	0.3	1.90	3	2.722
2	6.5	1.2	3.0	2.3		2	2.442	0.3	2.07	3	2.736
3	7.0	1.2	3.0	2.8		2	2.512	0.3	2.23	3	2.796
4	7.5	0.3	2.7	2.5	2.0	3	2.876	0.3	2.40	3	2.866
5	8.0	0.3	2.7	3.0	2.0	3	2.946	0.3	2.57	3	2.936
6	8.5	0.3	2.7	3.0	2.5	3	3.006	0.3	2.73	3	3.020
7	9.0	0.3	3.0	2.7	3.0	3	3.076	0.3	2.90	3	3.076
8	9.5	1.2	3.0	3.0	2.3	3	3.123	0.3	2.30	4	3.407
9	10.0	1.2	3.0	3.0	2.8	3	3.193	0.3	2.43	4	3.477

Table 4.5: Comparison of results obtained for the proposed integer programming model and Shin and Joo's model for modified finished pass range at $T_i = 25$ min (Example 2).

		DP model var dia						DP model constant dia					
S.No	d_t mm	d_0^{opt} mm	d_1^{opt} mm	d_2^{opt} mm	d_3^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece	d_0^{opt} mm	d_1^{opt} mm	d_2^{opt} mm	d_3^{opt} mm	n^{opt}	C_{to}^{opt} \$/piece
1	6.0	1.2	2.4	2.4		2	2.234	1.2	2.1	2.7		2	2.372
2	6.5	1.2	2.7	2.6		2	2.286	1.2	3.0	2.3		2	2.442
3	7.0	1.2	2.9	2.9		2	2.337	1.2	3.0	2.8		2	2.512
4	7.5	1.2	2.1	2.1	2.1	3	2.629	1.2	2.1	2.1	2.1	3	2.843
5	8.0	1.2	2.3	2.3	2.3	3	2.676	1.2	2.6	2.1	2.1	3	2.913
6	8.5	1.2	2.5	2.4	2.4	3	2.722	1.2	3.0	2.2	2.1	3	3.020
7	9.0	1.2	2.6	2.6	2.6	3	3.676	1.2	3.0	2.7	2.1	3	3.053
8	9.5	1.2	2.8	2.8	2.7	3	2.810	1.2	3.0	3.0	2.3	3	3.123
9	10.0	1.2	3.0	2.9	2.9	3	2.853	1.2	3.0	3.0	2.8	3	3.193

Table 4.6: Comparison of results obtained for dynamic programming model using variable and constant diameters for modified finish pass range at $T_i = 25$ min (Example 3).

S.No.	d_{0q} mm	v^*_{0q} m/min	f^*_{0q} mm/rev	T^*_{0q} min	$C^{*'}_{tovar\ 0q}$ \$/piece 10^{-3}	$C^{*''}_{tovar\ 0q}$ \$/piece 10^{-5}	C^*_{to0q} \$/piece
0	0.3	143.9	0.853	15.78	5.437	1.813	0.515
1	0.4	133.04	0.709	19.01	6.702	2.234	0.517
2	0.5	174.7	0.614	21.98	7.920	2.640	0.518
3	0.6	172.6	0.546	24.74	9.102	3.034	0.519
4	0.7	170.7	0.494	27.36	10.026	3.419	0.520
5	0.8	168.9	0.454	29.83	11.388	3.796	0.521
6	0.9	167.3	0.421	32.21	12.500	4.167	0.523
7	1.0	165.8	0.393	34.49	13.598	4.533	0.524
8	1.1	164.4	0.370	36.68	14.680	4.894	0.525
9	1.2	163.0	0.350	38.85	15.750	5.252	0.526

Table 4.7: Optimal cutting conditions and costs when the total stock is removed in a single finish pass (Example 4).

S.No.	d_{iq} mm	v^*_{iq} m/min	f^*_{iq} mm/rev	T^*_{iq} min	$C^{*'}_{tovar\ iq}$ \$/piece 10^{-3}	$C^{*''}_{tovar\ iq}$ \$/piece 10^{-5}	C^*_{toiq} \$/piece
0	1.0	84.60	0.51	59.97	12.105	4.035	0.522
1	1.1	84.50	0.466	59.97	13.264	4.421	0.523
2	1.2	84.45	0.429	59.97	14.419	4.807	0.524
3	1.3	84.39	0.397	59.97	15.570	5.190	0.526
4	1.4	84.33	0.370	59.97	16.717	5.572	0.527
5	1.5	84.27	0.347	59.97	17.861	5.954	0.528
6	1.6	84.22	0.326	59.97	19.00	6.334	0.529
7	1.7	84.16	0.308	59.97	20.141	6.714	0.530
8	1.8	84.13	0.292	59.97	21.28	7.092	0.531
9	1.9	84.08	0.277	59.97	22.401	7.470	0.532
10	2.0	84.04	0.264	59.97	23.540	7.846	0.534
11	2.1	84.01	0.252	59.97	24.670	8.222	0.535
12	2.2	83.97	0.241	59.97	25.790	8.598	0.536
13	2.3	83.93	0.231	59.97	26.917	8.972	0.537
14	2.4	83.89	0.222	59.97	28.039	9.346	0.538
15	2.5	83.87	0.214	59.97	29.159	9.720	0.539
16	2.6	83.84	0.206	59.97	30.277	10.092	0.540
17	2.7	83.81	0.199	59.97	31.394	10.465	0.541
18	2.8	83.77	0.192	59.97	32.508	10.836	0.543
19	2.9	83.75	0.185	59.97	33.622	11.207	0.544
20	3.0	83.72	0.180	59.97	34.73	11.578	0.545

Table 4.8: Optimal cutting conditions and costs when the total stock is removed in a single rough pass (Example 4).

S.No.	Characterstics	SUMT DFP	GRG	SQP	DP using single pass
1	Min. prod. time	10.467	10.391	10.37	9.08
2	Real time User time System time	1.3 0.8 0.0	1.0 0.4 0.0	0.2 0.1 0.0	1.1 0.9 0.0
3	Feasible initial point required	Yes	Yes	No	No
	Initial speed	170.0 170.0 170.0	170.0 170.0 170.0	170.0 170.0 170.0	– – –
	Feed	0.25 0.25 0.25	0.25 0.25 0.25	0.25 0.25 0.25	– – –
	Depth of cut	1.33 1.33 1.34	1.33 1.33 1.34	1.33 1.33 1.34	– – –
4	Optimal speed	212.04 211.23 211.28	212.0 212.0 212.0	212.0 212.0 212.0	212.0 212.0
	Feed	0.3487 .3480 .3486	0.3499 0.3499 0.3499	0.3499 0.3499 0.3499	0.2512 0.2512
	Depth of cut	1.449 1.3499 1.1977	1.3455 1.3248 1.3297	1.55 1.45 1.00	2.0 2.0
5	No. of passes required for problem formulation	Yes	Yes	Yes	No
6	Passes taken to assess comp. time	3	3	3	Not required
	Optimal passes obtained directly	No	No	No	Yes (2)
	Procedure to obtain optimal number of passes	All possible no. of passes (2,3,4)	All possible no. of passes (2,3,4)	Treating no. of pass continuous variable and solving two more problems arround continuous no. of passes 2.58, (2,3)	Uniquely obtained

Table 4.9: Optimal cutting conditions, and costs and times for various nonlinear techniques and dynamic programming method (Example 5).

Chapter 5

Turning of Stepped Shafts

5.1 Introduction

In the previous chapters the emphasis has been on optimization of single as well as multipass straight turning operation. It has been found in section 4.3 that DP approach does not require the knowledge of optimal number of passes in order to make multipass turning model. Instead the optimal number of passes are uniquely determined using DP approach which requires repetitive single pass solutions. The single pass solution methodology discussed in section 3.2 has advantage over other techniques that the optimal machining conditions are determined without the knowledge of feasible/nonfeasible starting point and without experimentation with optimization parameters. Thus, DP alongwith the single pass solution methodology discussed in the present work has its own advantages and has been selected to find the optimal clubbing of machinable volumes in vertical, horizontal or circular manner for stepped shafts. Figure 5.1 shows an example of rough turning of a stepped shaft. The various machinable volume segments (1), (2) and (3) have been obtained by drawing horizontal and vertical lines from the ends of cylindrical volumes. These volumes are defined as machinable volume, seed volume, and compound volumes.

Machinable Volume

The smallest volume segment enclosed by two consecutive horizontal and two consecutive vertical lines is called the machinable volume. In Fig. 5.1, (1), (2) and (3) are machinable volumes.

Seed Volume

It is a machinable volume which has at least one surface as machined boundary of shaft. Volumes (1) and (5) are seed volumes.

Compound Volume

This is a volume consisting of a seed and a few non-seed volumes, which can be machined together. For example volumes (1,3) and volumes (2,5) are compound volumes. In order to make feasible sets of compound volumes a seed volume is selected. Starting with the seed volume, say (5), first vertical clubbing is performed and volumes (5,6) and (5,6,7)

are generated. Thereafter, horizontal clubbing of volumes (5,2) is performed. The circular clubbing is generated from small to larger compound volumes (5,2,3,6) and (5,2,3,4,6,7).

It is interesting to note that grouping of volumes (1,2) and (5) volumes (5,2) and (1) are two alternative approaches for machining. These alternative approaches will give different minimum production costs. Therefore it becomes an interesting decision to select optimal grouping of machinable volumes so that the minimum production cost is obtained. For all seed and compound volumes, the optimal number of passes has to be determined for optimal grouping of machinable volumes to machine the shaft.

The horizontal grouping of machinable volumes has been considered only in recent literature for turning of rough stepped shafts [Jha, 1996; Prasad et al., 1997]. In a similar work for the milling process, the machinable volumes have been grouped using a heuristic based on ascending machining cost per unit volume [Yellowley and Fisher, 1994]. A linear integer model has been suggested in this chapter to find the minimum production cost and optimal grouping of machinable volumes for stepped shafts. It has been verified that the suggested model gives better minimum production cost than that obtained using heuristic method or common practice.

5.2 Optimal Grouping of Machinable Volumes

In order to make optimal grouping of machinable volumes, all seed volumes are recognized and compound volumes corresponding to all seed volumes are generated. These seed and compound volumes are candidates to be considered for grouping. Let a candidate volume be represented as V_g and the associated minimum production cost obtained using DP alongwith single pass methodology be C_{tog}. The candidate volume is either a seed or a compound volume. A matrix **A** is defined to represent the composition of machining volumes such that a_{eg} is one if machinable volume e belongs to candidate volume g and is zero otherwise. The minimization of production cost for turning of a stepped shaft is defined as

minimize

$$\sum_g C_{tog} xg \tag{5.1}$$

where xg is a binary variable. Its value is one if g^{th} volume is selected in optimal grouping and is zero otherwise, subject to

$$\sum_g a_{eg} xg \geq 1 \quad \text{for all } e \tag{5.2}$$

The above integer programming formulation is solved using LINDO optimizer.

5.2.1 Example 1

A stepped shaft shown in Fig. 5.1. has six cylindrical faces. The manufacturing of this shaft has been performed with a blank. It may be noted that surfaces (i), (ii) and (iii) can be machined while tool approaches from the right and surfaces (v) and (vi) can be machined while tool approaches from the left side. Surface (iv) can be machined from tool approaching from either side.

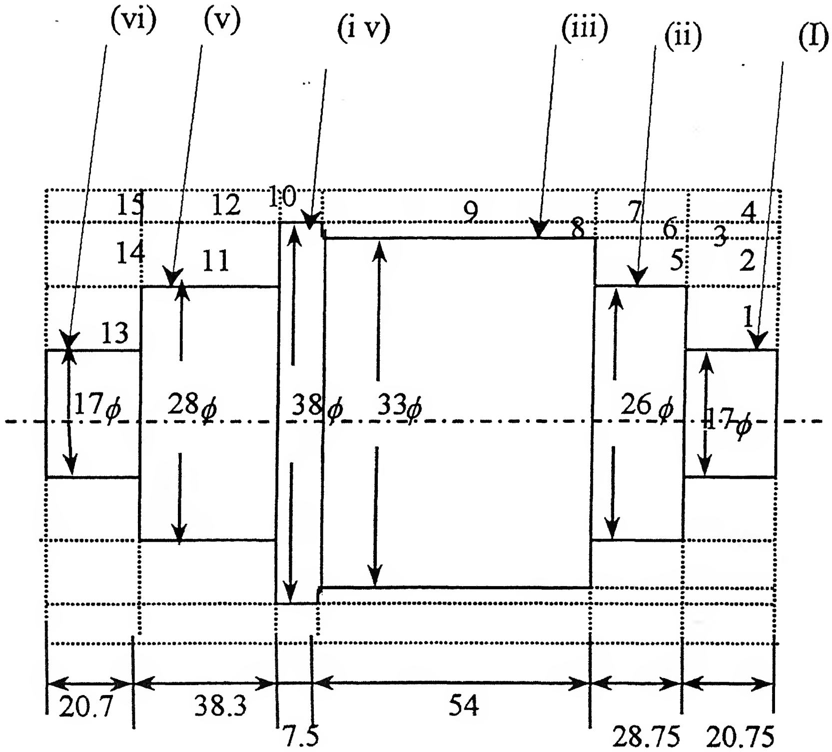

Figure 5.1: Rough turning of a stepped shaft from 40 mm diameter blank

In turning of stepped shafts, various small volume segments can be formed by drawing vertical lines from the ends of different cylindrical surfaces and horizontal lines from ends of various facing surfaces. The various volume segments thus formed are denoted from 1 to 15. The following are the example data:

- Blank diameter = 40 mm.
- Allowable cutting speed range, $v_{i\ min} = 5$ m/min; $v_{i\ max} = 500$ m/min.
- Allowable cutting feed range, $f_{i\ min} = 0.1$ mm/rev; $f_{i\ max} = 0.9$ mm/rev.
- Allowable cutting depth range, $d_{i\ min} = 1.0$ mm; $d_{i\ max} = 3.0$ mm.

- Constant for tool life equation, $C_{Ti}^{5} = 6 \times 10^{11}$.
- Exponents for tool life equation, $\alpha_{0i} = 5.0$; $\beta_{0i} = 1.75$; $\gamma_{0i} = 0.75$.
- Allowable cutting tool life range, $T_{i\ min} = 25$ min; $T_{i\ max} = 45$ min.
- Constant for cutting force equation, $K_{1i}' = 1.059$
- Exponents for cutting force equation, $\alpha_{1i} = 0$, $\beta_{1i} = 0.75$; $\gamma_{1i} = 0.95$.
- Maximum allowable cutting force, $F_{i\ max} = 1.962$ kN.
- Cutting power equation constant, $K_{2i}' = 0.02076$.
- Exponents for cutting power equation, $\alpha_{2i} = 1.0$, $\beta_{2i} = 0.75$; $\gamma_{2i} = 0.95$.
- Maximum allowable cutting power, $P_{i\ max} = 5$ kW.
- Constant for surface finish equation, $K_{3i}' = 0.125/r$; nose radius $r = 1.2$ mm.
- Exponents for surface finish equation, $\beta_{3i} = 2.0$.
- Maximum allowable surface roughness, $R_{i\ max} = 100\ \mu$m.
- Constants for tool approach and detraction time, $h_{1i} = 7 \times 10^{-4}$; $h_{2i} = 0.3$ min.
- Time components, $t_{p1} = 0.75$ min/piece; $t_{ci} = 1.5$ min/cutting edge.
- Cost components, $k_{li} = \$\ 0.5$/min; $k_{ti} = \$\ 2.5$/cutting edge.

For the data, the constants calculated using eqn. (3.4) are:
$A0_{1i} = 0.249$, assuming $T_i = 25$ min (test point)
$k_{li}t_{p1} = 0.375$

Solution Procedure

There are six seed volumes in this example. These are volumes (1), (5), (8), (10), (11) and (13). The compound volumes corresponding to each of these seed volumes are given in Table 5.1. The initial and final diameters and lengths of steps, minimum production cost, volume and minimum production cost/volume are also given in the same table. There are 15 machinable volumes and 35 candidate volumes. The minimum production cost for each candidate volume have been obtained using DP alongwith single pass solution methodology and are given in Table 5.1. The integer programming model formulated to obtain the minimum production cost as well as optimal grouping of machinable volumes is

minimize

$0.3356x1 + 0.5154x2 + 0.6913x3 + 0.6999x4 + 0.3546x5 + 0.3772x6 + 0.5479x7 + 0.3940x8 + 0.4330x9 + 0.6185x10 + 0.2177x11 + 0.4172x12 + 0.2537x13 + 0.2797x14 + 0.4796x15 + 0.5246x16 + 0.1580x17 + 0.2153x18 + 0.2458x19 + 0.2678x20 + 0.1986x21 + 0.2206x22 + 0.2559x23 + 0.2589x24 + 0.2809x25 + 0.2779x26 + 0.3084x27 + 0.3305x28 + 0.3914x29 + 0.4072x30 + 0.4408x31 + 0.4652x32 + 0.3417x33 + 0.6911x34 + 0.6997x35$

subject to

$x1 + x2 + x3 + x4 >= 1$

$x2 + x3 + x4 + x8 + x9 + x10 >= 1$

$x3 + x4 + x9 + x10 + x14 + x16 >= 1$

$x4 + x10 + x16 + x20 + x25 + x28 >= 1$

$x5 + x6 + x7 + x9 + x10 >= 1$

$x6 + x7 + x9 + x10 + x13 + x14 + x15 + x16 >= 1$

$x7 + x10 + x15 + x16 + x19 + x20 + x24 + x25 + x27 + x28 >= 1$

$x11 + x12 + x13 + x14 + x15 + x16 >= 1$

$x12 + x15 + x16 + x18 + x19 + x20 + x23 + x24 + x25 + x26 + x27 + x28 >= 1$

$x17 + x18 + x19 + x20 + x21 + x22 + x23 + x24 + x25 + x26 + x27 + x28 >= 1$

$x29 + x30 + x31 + x32 >= 1$

$x21 + x22 + x23 + x24 + x25 + x26 + x27 + x28 + x30 + x32 >= 1$

$x33 + x34 + x35 >= 1$

$x31 + x32 + x34 + x35 >= 1$

$x22 + x26 + x27 + x28 + x32 + x35 >= 1$

END

The solution to this linear integer programming problem has been obtained using LINDO software and is given in Table 5.3. The minimum production cost turns out to be \$ 2.061/piece. It is lower than that obtained using minimum production cost/volume heuristic (\$ 2.084/piece) and that obtained by adopting and the common practice of horizontal clubbing (\$ 2.1223/piece). Thus there is significant improvement in production cost by using the suggested model which yields the minimum cost.

5.2.2 Example 2

In this second example the geometry of stepped shaft is similar to that of the first example but the shaft dimensions are different. The shaft is shown in Fig. 5.2.

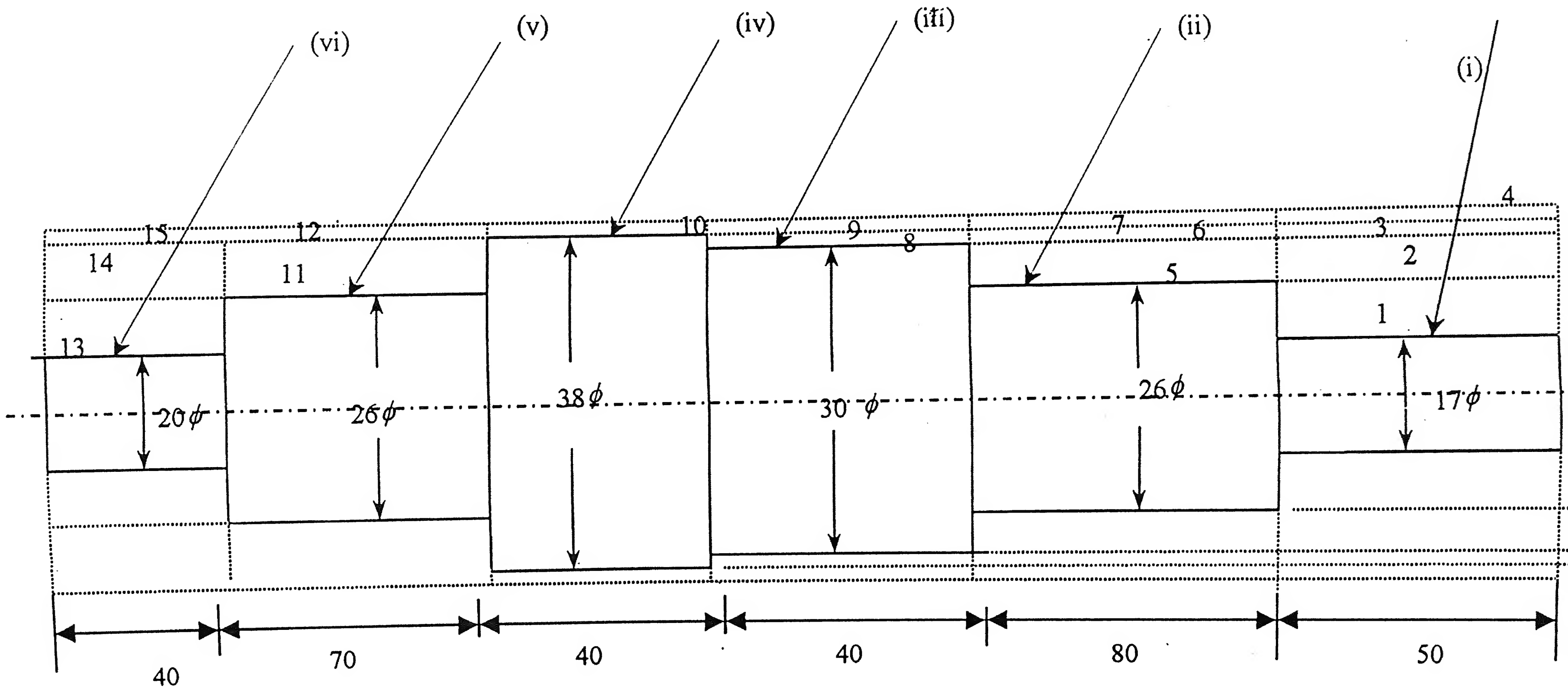

Figure 5.2: Rough turning of a stepped shaft from 42 mm diameter blank

The mathematical model has been built on the basis of the same 15 machinable volumes and 35 candidate volumes. The minimum production cost for each candidate volume and other calculations are presented in Table 5.2. The minimization problem now gives

minimize

$0.3859x1+0.5821x2+0.8201x3+1.0275x4+0.2254x5+0.5149x6+0.7566x7+0.2725x8+0.6492x9+0.9582x10+0.3848x11+0.4165x12+0.5545x13+0.6606x14+0.6495x15+0.7952x16+0.1972x17+0.2443x18+0.3386x19+0.3976x20+0.2797x21+0.3268x22+0.3268x23+0.4212x24+0.4801x25+0.3740x26+0.4683x27+0.5273x28+0.4880x29+0.7183x30+0.5955x31+0.8716x32+0.1935x33+0.6009x34+0.7967x35$

subject to the similar 15 constraints as for example 1 because the shaft geometries are similar and the same candidate volumes have been selected in both cases. The minimum production cost for the above problem turns out to be \$ 2.5694/piece. It is important to note that the grouping of candidate volumes is different in this case even though the shafts are similar. Incidentally for this case the production cost using heuristic method and the common practice of horizontal clubbing turn out to be the same, i.e., \$ 2.6353/piece.

5.3 Remarks

To sum up the mathematical model suggested in this chapter successfully gives the minimum production cost for rough turning of stepped shafts through optimal grouping of machinable volumes. The optimal grouping of machinable volumes achieved by mathematical model yields better minimum production cost than that obtained by using the common practice of clubbing these volumes horizontally or using heuristic method based on minimum production cost/volume. Similar stepped shafts may have different optimal grouping which depends on the size and location of machinable volumes.

Sl. No.	Seed Vol.	Compound Volume	D_0 mm	D_i mm	d_{t1} mm	L mm	C_{to} \$/piece	Volume cu mm	Cost/vol \$ $10^{-5}/mm^3$
1	1	1	26	17	4.5	20.75	0.3356	6125	5.4796
2		1,2	33	17	8.0	20.75	0.5154	13038	3.9532
3		1,2,3	38	17	10.5	20.75	0.6913	18823	3.6726
4		1,2,3,4	40	17	11.5	20.75	0.6999	21365	3.276
5	5	5	33	26	3.5	28.75	0.3546	9326	3.8024
6		5,6	38	26	6.0	28.75	0.3772	17342	2.1751
7		5,6,7	40	26	7.0	28.75	0.5479	20864	2.6260
8		5,2	33	26	3.5	49.5	0.3940	16056	2.45386
9		5,2,3,6	38	26	6.0	49.5	0.4330	29858	1.4502
10		5,2,3,4,6,7	40	26	7.0	49.5	0.6185	35923	1.7218
11	8	8	38	33	2.5	54.0	0.2177	5056	1.4459
12		8,9	40	33	3.5	54.0	0.4172	39188	1.0646
13		8,6	38	33	2.5	82.75	0.2537	23069	1.0997
14		8,6,3	38	33	2.5	103.5	0.2797	28854	0.0969
15		8,6,7,9	40	33	3.5	82.75	0.4796	33210	1.0646
16		8,6,3,4,7,9	40	33	3.5	103.5	0.5246	41539	1.2629
17	10	10	40	38	1.0	7.5	0.1580	919	7.1944
18		10,9	40	38	1.0	61.5	0.2153	7533	2.8573
19		10,9,7	40	38	1.0	90.25	0.2458	11058	2.2229
20		10,9,7,4	40	38	1.0	111.0	0.2678	13600	1.9691
21		10,12	40	38	1.0	45.8	0.1986	5612	3.5392
22		10,12,15	40	38	1.0	66.5	0.2206	8148	2.7075
23		10,9,12	40	38	1.0	99.8	0.2559	12228	2.0928
24		10,9,7,12	40	38	1.0	102.55	0.2589	12565	2.0606
25		10,9,7,4,12	40	38	1.0	123.3	0.2809	15107	1.8594
26		10,9,12,15	40	38	1.0	120.5	0.2779	14763	1.8823
27		10,9,7,12,15	40	38	1.0	149.25	0.3084	18286	1.6865
28		10,9,7,4,12,15	40	38	1.0	170.0	0.3305	20829	1.5868
29	11	11	38	28	5.0	38.3	0.3914	19853	1.9715
30		11,12	40	28	6.0	38.3	0.4072	24546	1.6589
31		11,14	38	28	5.0	59.0	0.4408	30583	1.4413
32		11,12,15,14	40	28	6.0	59.0	0.4652	37812	1.2303
33	13	13	28	17	5.5	20.7	0.3417	8048	4.2459
34		13,14	38	17	10.5	20.7	0.6911	18778	3.6804
35		13,14,15	40	17	11.5	20.7	0.6997	21314	3.2828

Table 5.1: Minimum production cost and production cost/volume for compound volume.

Sl. No.	Seed Vol.	Compound volume	D_0 mm	D_i mm	d_{t1} mm	L mm	C_{to} \$/piece	Volume cu mm	Cost/vol \$ $10^{-5}/mm^3$
1	1	1	26	17	4.5	50.0	0.3859	15197	2.5394
2		1,2	30	17	6.5	50.0	0.5821	23994	2.4259
3		1,2,3	38	17	10.5	50.0	0.8201	45357	1.8081
4		1,2,3,4	42	17	12.5	50.0	1.0275	57923	1.7790
5	5	5	30	26	2.0	80.0	0.2254	14074	1.6012
6		5,6	38	26	4.0	80.0	0.5149	48255	1.0670
7		5,6,7	42	26	8.0	80.0	0.7566	68361	1.1068
8		5,2	30	26	2.0	130.0	0.2725	22871	1.1914
9		5,2,3,6	38	26	4.0	130.0	0.6492	78414	0.8279
10		5,2,3,4,6,7	42	26	8.0	130.0	0.9582	111087	0.8536
11	8	8	38	30	4.0	40.0	0.3848	17090	2.2518
12		8,9	42	30	6.0	40.0	0.4165	27143	1.5345
13		8,6	38	30	4.0	120.0	0.5545	51271	1.0816
14		8,6,3	38	30	4.0	170.0	0.6606	72633	0.9095
15		8,6,7,9	42	30	6.0	120.0	0.6495	81431	0.7977
16		8,6,3,4,7,9	42	30	8.0	170.0	0.7952	115359	0.6893
17	10	10	42	38	2.0	40.0	0.1972	10053	1.9612
18		10,9	42	38	2.0	80.0	0.2443	20106	1.2151
19		10,9,7	42	38	2.0	160.0	0.3386	40212	0.8421
20		10,9,7,4	42	38	2.0	210.0	0.3976	52779	0.7533
21		10,12	42	38	2.0	110.0	0.2797	27646	1.0117
22		10,12,15	42	38	2.0	150.0	0.3268	37699	0.8670
23		10,9,12	42	38	2.0	150.0	0.3268	57805	0.8670
24		10,9,7,12	42	38	2.0	230.0	0.4212	70371	0.7286
25		10,9,7,4,12	42	38	2.0	280.0	0.4801	47752	0.6823
26		10,9,12,15	42	38	2.0	190.0	0.3740	67858	0.7832
27		10,9,7,12,15	42	38	2.0	270.0	0.4683	80424	0.6901
28		10,9,7,4,12,15	42	38	2.0	320.0	0.5273	42223	0.6557
29	11	11	38	26	4.0	70.0	0.4880	59816	1.1558
30		11,12	42	26	8.0	70.0	0.7183	66350	1.2005
31		11,14	38	26	4.0	110.0	0.5955	93996	0.8974
32		11,12,15,14	42	26	8.0	110.0	0.8716	8671	0.9273
33	13	13	26	20	3.0	40.0	0.1935	32798	2.2316
34		13,14	28	20	9.0	40.0	0.6009	18778	1.8321
35		13,14,15	42	20	11.0	40.0	0.7967	21314	1.8592

Table 5.2: Minimum production cost and production cost/volume for compound volumes.

Sl. No.	Compound volume	Integer prog.	Compound volume	Min cost/ vol $ 10^{-5}$	Min cost \$/piece	Compound volume	Min cost \$/piece
1	1	0.3356	8,6,3	0.0969	0.2797	8,6,3	0.2797
2	5,2,3,6	0.4330	11,12,15,14	1.2303	0.4652	12,15,10,19,7,4	0.3305
3	8	0.2177	10,9,7,4	1.9691	0.2678	11,14	0.4408
4	10,9,7,4	0.2678	2,5	2.4539	0.3940	2,5	0.3940
5	11,12,15,14	0.4652	13	4.2459	0.3417	13	0.3417
6	13	0.3417	1	5.4796	0.3356	1	0.3356
	total min cost/piece	2.061 math. model			2.084 heuristic method		2.1223 common practice

Table 5.3: Comparison of minimum production cost for the proposed method and common practice for stepped turning

Sl. No.	Compound volume	Integer prog.	Compound volume	Min cost/ vol $ 10^{-5}$	Min cost \$/piece	Compound volume	Min cost \$/piece
1	8,6,3,4,7,9	0.7952	10,9.7,4,12,15	0.6557	0.5273	8,6,3	0.6606
2	11,14	0.5955	8,6,3	0.7977	0.6606	12,15,10,9,7,4	0.5273
3	10,12,15	0.3268	11,14	0.8974	0.5955	11,14	0.5955
4	2,5	0.2725	2,5	1.1914	0.2725	2,5	0.2725
5	13	0.1935	13 .	2.2316	0.1913	13	0.1913
6	1	0.3859	1	2.5394	0.3859	1	0.3859
	total min cost/piece	2.5694 math. model			2.6353 heuristic method		2.6353 common practice

Table 5.4: Comparison of minimum production cost for the proposed method and common practice for stepped turning

Chapter 6

Conclusions and Scope for Future Work

6.1 Conclusions

The multipass turning problem has been solved using results of single pass turning problems. The three basic methods used to solve the single pass problem are:
(i) Differential calculus with graphical search over nomographs of the constraints.
(ii) Nonlinear programming (NLP) methods such as SUMT with DFP, GRG, requiring initial feasible points and methods which may be started from any arbitrary starting point, for example, SQP method, or which do not require any starting point such as some of the geometric programming (GP) methods.
(iii) One dimensional search methods over tool life and finding the maximum permissible feed under the constraints for depth of cut.

The last method has been selected due to simplicity in understanding and implementation. The method requires simple calculations if the constraints are monomials and it does not require any starting point. The parametric analysis single pass solution methodology has also been performed. It has been observed that optimal tool life cutting speed and feed do not change with workpiece dimensions for turning operations provided the depth of cut and the cutting environment remain the same. The minimum production cost or time is obtained easily using simple calculations for changed workpiece dimensions.

It has been a common practice to remove a small depth in the finish pass and remaining depth of cut equally divided among all rough passes. An integer programming model has been developed on the basis of equal workpiece diameter. The model contains one finish pass and assumed maximum number of rough passes. It makes use of the production costs obtained for feasible series of depth of cut for finish as well as rough passes assuming some least count for depth of cut accuracy. The model has been solved using LINDO software. The optimal number of passes are also determined alongwith optimal depth of cut distribution. For each pass, depending upon the optimal depth of cut, optimal speed

integer programming model is logical selection of small depth of cut for finish pass alongwith optimal number of passes which minimize the production cost per piece.

It has been observed that the rough passes need not necessarily be of equal depth at minimum production cost. This view has also been confirmed after solution of a few examples using integer programming model. It has also been observed that sometimes LINDO software is not able to achieve or terminate at optimal results. Therefore, a second model based on dynamic programming (DP) has been developed. Each pass represents a stage and each stage problem can be solved using any single pass solution methodology. Single pass problems are required to be repeatedly solved for a given depth of cut in each stage pass. The single stage/pass problem for the same depth of cut need not to be solved again and again if the single pass solution methodology suggested in the present work is used. The solution to new workpiece diameter and for the same depth of cut can easily be computed using parametric analysis. It has been observed that in cases where LINDO is not able to achieve optimal results the dynamic programming approach clearly determines the optimal solution.

The computation times of three nonlinear methods viz. SUMT with DFP, GRG, SQP and dynamic programming approach suggested in the present work have been compared for an example. The sequential quadratic programming approach treats all the passes simultaneously. The computer code-NCONF for this method has been obtained from IMSL library and it may not be available for all PC users easily. This NLP method requires approximately the same order of computation time as is required for dynamic programming approach. The first two methods essentially require feasible starting point and considerably large computation time in comparison with DP approach alongwith single pass solution methodology.

In the determination of optimal clubbing of volumes for rough turning of a stepped shaft, the common practice has been to club volumes horizontally only. In a similar work for milling, machinable volumes have been clubbed using a heuristic of minimum production cost per unit volume. The optimal clubbing of machinable volumes can be obtained using an integer programming model suggested in Chapter 5 for this purpose. First the feasible sets of compound volumes are formed and the corresponding minimum production costs are obtained using DP method. Then these sets are chosen such that all machinable volumes are covered and minimum production cost is obtained.

To sum up there are following two approaches to solve multipass turning model : (i) solve all passes simultaneously and (ii) synthesize multipass solution using single pass solutions. The later one has been adopted in the present work because it decomposes complex multipass problems into simple single pass problems. The dynamic programming model has been used for this purpose. Out of various single pass solution methods viz. non-linear

programming techniques and graphical search over nomographs of constraints and chip-breaking region to find the optimal machining conditions, a simple one dimensional search over tool life has been selected to find the optimal machining conditions. At a given depth of cut, an optimal tool life can be located at which minimum production cost is obtained using the maximum permissible feed under the constraints. The optimal cutting tool life, speed and feed will remain the same for changed workpiece dimensions if the depth of cut and cutting environment is not changed. Dynamic programming alongwith single pass solution methodology has emerged as a good solution technique to obtained optimal cutting conditions for multipass turning with monomial constraints.

6.2 Scope

DP approach alongwith single pass solution methodology does not requires knowledge of any starting point. It also determines the optimal number of passes uniquely and is easy to implement. This may act as cutting conditions optimization module in computer assisted process planning of turning operations. The present work may be easily extended to obtain optimal machining conditions for plane and face milling operations where cutting tool life is similar to extended Taylor's tool life equation and constraints are monomial. It will also interesting to obtain the optimal machining conditions for continuous profile shafts using single pass methodology.

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Appendix A

SUMT with DFP Method

Sequential Unconstrained Minimization Technique (SUMT) are penalty function methods which transform basic optimization problem into alternative optimization formulation such that the numerical solutions are obtained by solving a sequence of unconstrained minimization problems. Let the original optimization problem be stated as.

Find decision vector $\mathbf{X}$ which minimizes

$$f_0(\mathbf{X}) \tag{A.1}$$

subject to :

Equality constraints

$$f_i(\mathbf{X}) = 0 \tag{A.2}$$

where $i = 1 \ldots m_1$

and inequality constraints

$$f_i(\mathbf{X}) \leq 0 \tag{A.3}$$

where $i = m_1 \ldots m$

The above problem is converted to the following unconstrained optimization problem using interior penalty function method:

minimize

$$\phi(\mathbf{X}, r_j) = f_0(\mathbf{X}) + \sum_{i=1}^{i=m_1} \frac{1}{\sqrt{r_j}}(f_i(\mathbf{X}))^2 - \sum_{i=m_1}^{i=m} r_k \frac{1}{f_i(\mathbf{X})} \tag{A.4}$$

If the minimization of ϕ function is performed repeatedly for a sequence of values of penalty parameter r_j, the solution may be brought to converge to that of the original problem. The minimization of the ϕ function has been performed using Davidon Fletcher Powell's method discussed in the next section.

A.1 Davidon, Fletcher and Powell's Method

The iterative procedure of this method can be stated as follows.

Step 1: Start with an initial point $\mathbf{X_1}$ and a $n \times n$ positive definite matrix $\mathbf{H_1}$. Usually $\mathbf{H_1}$ is taken to be identity matrix $\mathbf{I}$. Set iteration number $k = 1$.

Step 2: Compute gradient of the function ∇f_k, at point $\mathbf{X_k}$ and set

$$\mathbf{S_k} = -\mathbf{H_k} \nabla f_k \tag{A.5}$$

Step 3: Find the optimal step length λ_k^* in the direction of $\mathbf{S_k}$ and set

$$\mathbf{X_{k+1}} = \mathbf{X_k} + \lambda_k^* \mathbf{S_k} \tag{A.6}$$

Step 4: Test the new point for optimality. If $\mathbf{X_{k+1}}$ is optimal terminate the iterative process. Else, go to *Step 5*.

Step 5: Update matrix $\mathbf{H_k}$ as

$$\mathbf{H_{k+1}} = \mathbf{H_k} + \mathbf{M_k} + \mathbf{N_k} \tag{A.7}$$

where

$$\mathbf{M_k} = \lambda_k^* \frac{\mathbf{S_k S_k^T}}{\mathbf{S_k^T Q_k}} \tag{A.8}$$

$$\mathbf{N_k} = -\frac{(\mathbf{H_k Q_k})(\mathbf{H_k Q_k})^\mathbf{T}}{\mathbf{Q_k^T H_k Q_k}} \tag{A.9}$$

and

$$\mathbf{Q}_k = \nabla f(\mathbf{X_{k+1}}) - \nabla f(\mathbf{X_k}) \tag{A.10}$$

Step 6: Set the new iteration number $k = k + 1$, and go to *Step 2*.

Appendix B

Generalized Reduced Gradient Method

Let the original problem be stated as

minimize

$$f_0 = f_0(\mathbf{X}) \tag{B.1}$$

subject to:

equality constraints

$$\mathbf{f}(\mathbf{X}) = \mathbf{0} \tag{B.2}$$

and bounds

$$\mathbf{a} \leq \mathbf{X} \leq \mathbf{b} \tag{B.3}$$

where $\mathbf{X}, \mathbf{a}, \mathbf{b}$ are N dimensional column vectors, $\mathbf{f}(\mathbf{X})$, is for any $\mathbf{X}$ under consideration, an m dimensional column vector and $f_0(\mathbf{X})$ is a real number. The f_0 and $\mathbf{f}$ are assumed continuously differentiable under the parallelotope defined by (B.3). The gradient of $f_0(\mathbf{X})$ is a row vector denoted by $f_0^{'}(\mathbf{X})$. Similarly, gradient of $f_i(\mathbf{X}), i = 1 \ldots m$ is a row vector denoted by $f_i^{'}(\mathbf{X})$. The notation $f^{'}(\mathbf{X})$ designates $(m \times n)$ matrix, the rows of which are $f_i(\mathbf{X}), i = 1 \ldots m$.

Vector $\mathbf{X}$ is partitioned into $\mathbf{x}$ and $\mathbf{y}$, where $\mathbf{y}$ is m dimensional and $\mathbf{x}$ is n dimensional with $n = N - m$. Hereafter, $\mathbf{y}$ is called vector of basic or dependent variables and $\mathbf{x}$ is called vector of non-basic or independent variables.

Notations $(\partial f_0/\partial \mathbf{x})$ and $(\partial f_0/\partial \mathbf{y})$ are called row vectors of f_0 with respect to $\mathbf{x}$ and $\mathbf{y}$. Similarly, $(m \times N)$ matrix $f^{'}(\mathbf{X})$ is partitioned into $(m \times n)$ matrix $(\partial \mathbf{f}/\partial \mathbf{x})$ and $(m \times m)$ matrix $(\partial \mathbf{f}/\partial \mathbf{y})$.

Let a feasible point be partitioned as to satisfy the following nondegeneracy assumption. There exists a partition of $\mathbf{X}$ into $\mathbf{x}$ and $\mathbf{y}$ such that

$$a_j \leq y_j^0 \leq b_j \forall j \tag{B.4}$$

$(\partial \mathbf{f}/\partial \mathbf{y}^0)$ stands for $(\partial \mathbf{f}/\partial \mathbf{y})$ computed at $\mathbf{x}^0$ and $\mathbf{y}^0$.
The GRG algorithm can be summarized as follows:

Step 1: Compute $-\mathbf{h}^0$, the direction of move for independent variable $\mathbf{x}$, using the following substeps.
Step 1.1: Compute the reduced gradient

$$\mathbf{g}^0 = (\frac{\partial \mathrm{f}_0}{\partial \mathbf{x}^0}) - (\frac{\partial \mathrm{f}_0}{\partial \mathbf{y}^0})((\frac{\partial \mathbf{f}}{\partial \mathbf{y}^0})^{-1})(\frac{\partial \mathbf{f}}{\partial \mathbf{x}^0}) \tag{B.5}$$

Step 1.2: Compute the projected reduced gradient $(\mathbf{p}^0)$ given by $\forall j$,

$$p_j^0 = \begin{cases} 0 & \text{if } x_j^0 = a_j \text{ and } -g_j^0 < 0 \\ 0 & \text{if } x_j^0 = b_j \text{ and } -g_j^0 > 0 \\ g_j^0 & \text{otherwise} \end{cases}$$

Step 1.3: Compute $\mathbf{h}^0$, the modified projected reduced gradient, i.e., the (opposite) gradient direction of move for the independent variable $\mathbf{x}$. This direction may simply be $\mathbf{h}^0 = \mathbf{p}^0$. However, it has been modified by making all the components such that $p_j^0(x_j^0 - a_j)$ or $p_j^0(x_j^0 - b_j)$ is too small and equal to zero. It has been checked that after these modification scalar product $(\mathbf{g}^0) \times (\mathbf{h}^0)$ is positive.

Step 2: Compute a first value of positive number θ_1.

Step 3: Compute $\mathbf{x}^0 - \theta_1\mathbf{h}^0$, and project it onto parallelotope $a_j \leq x_j \leq b_j, \forall j$, to obtain $x^{\,1}$, i.e. set

$\forall j$,

$$x^{\,1} = \begin{cases} a_j & \textit{if} \text{ if } \mathbf{x}^0 - \theta_1\mathbf{h}^0 < a_j \\ b_j & \textit{if} \text{ if } \mathbf{x}^0 - \theta_1\mathbf{h}^0 > b_j \\ \text{if } \mathbf{x}^0 - \theta_1\mathbf{h}^0 \text{ otherwise} \end{cases}$$

Step 4: Compute a feasible $\mathbf{X}^1$ corresponding to θ_1, i.e., to solve with respect to $\mathbf{y}$ the system of m equations in m unknowns:

$$\mathbf{f}(\mathbf{x}^1, \mathbf{y}) = \mathbf{0} \tag{B.6}$$

This is done by some iterative method described after step 5.
Step 4.1: If no speedy convergence is achieved then decrease $\theta_1 = 0.5\theta_1$ and go to step 3, with same $\mathbf{h}^0$.
Step 4.2: Otherwise, let $\mathbf{y}^1$ be the solution obtained for eqns. (B.6) and $\mathbf{X}^1$ the corresponding point in N dimensional space.

Step 4.2.1: If $f_0(\mathbf{X}^1) \geq f_0(\mathbf{X}^0)$, then decrease θ_1 as above and go to step 3 with same $\mathbf{h}^0$.
Step 4.2.2: Otherwise, at the end of step 4, we have some feasible $\mathbf{X}^1$ which satisfies

$$f_0(\mathbf{X}^1) < f_0(\mathbf{X}^0) \tag{B.7}$$

Step 5: At this stage either set $\mathbf{X}^0 := \mathbf{X}^1$ and begin a new iteration, or try to improve the last obtained value for θ_1. In doing so step 3 is reached for any improved value of θ_1, with same h^0 and eventually terminate with some $\mathbf{X}^1$ satisfying eqns. (B.6) and then begin new iteration, $\mathbf{X}^0 := \mathbf{X}^1$.

B.1 Iterative Method and Change from Basic to Non-basic variables

The solution to eqns. (B.6) means a solution

$$|\,\mathbf{f}(\mathbf{x}^1, \mathbf{y})\,| = \epsilon \tag{B.8}$$

where ϵ is a given positive number (tolerance), and $|\,.\,|$ is some norm in m dimensional space. Let $\mathbf{y}^1, \mathbf{y}^2$ and so on upto $\mathbf{y}^t$ be successive iterates obtained in applying *step 4* with in the same iteration of GRG. The term 'no speedy convergence occurs' implies that eqns. (B.6) does not hold for any $t \leq t_0$. Consequently , the iterations in *step 4* are stopped as soon as eqns. (B.6) is satisfied for some $\mathbf{y}^t$.
Set $\mathbf{y}^1 = \mathbf{y}^t$, to obtain $\mathbf{X}^1$ in *step 4.2.* A pseudo-Newton method has been used in our code as

$$\mathbf{y}^{t+1} = \mathbf{y}^t - ((\frac{\partial \mathbf{f}}{\partial \mathbf{y}^0}))^{-1}\mathbf{f}(\mathbf{x}^{t+1}, \mathbf{y}^t) \tag{B.9}$$

If $\mathbf{X}^t = (\mathbf{x}^1, \mathbf{y}^t) \in P$ and $\mathbf{X}^{t+1} = (\mathbf{x}^1, \mathbf{y}^{t+1}) \ni P$, then intersection of segment $\mathbf{X}^t - \mathbf{X}^{t+1}$ with boundary of P is calculated. For some index r $\mathbf{y}_r^t = a_r$ or b_r. The basic variable $\mathbf{y}_r$ is changed with non-basic (independent) variable $\mathbf{x}_s$ for with some index s such that the following expression is maximized $\forall j$
min $(|\,\alpha_{rj}(b_j - x_j^0), |\,\alpha_{rj}(x_j^0 - a_j))$
where $|\,\alpha_{rj}$, is element of r^{th} row and j^{th} column of the matrix $(\frac{\partial \mathbf{f}}{\partial \mathbf{y}^0})^{-1}(\frac{\partial \mathbf{f}}{\partial \mathbf{x}^0})$.

Appendix C

Sequential Quadratic Programming Method

NCONF subroutine is based on subroutine NLPQL, a FORTRAN code developed by Schittkowaski [1986]. It uses a sequential quadratic programming method to solve the general nonlinear programming problem. Let the original optimization problem be stated as minimize

$$f_0(\mathbf{X}) \tag{C.1}$$

subject to :
equality constraints

$$f_i(\mathbf{X}) = 0 \tag{C.2}$$

where $i = 1 \ldots m_1$
and inequality constraints

$$f_i(\mathbf{X}) \geq 0 \tag{C.3}$$

where $i = m_1 \ldots m$. It should be noted that the code NCONF has been made to accept greater than or equal to form of the constraints. Therefore all the constraints in multipass turning problem which are in less than or equal to form have been modified accordingly and

$$\mathbf{a} \leq \mathbf{X} \leq \mathbf{b} \tag{C.4}$$

where all the functions involved in the problem have been assumed to be continuously differentiable. The method based on iterative formulation and solution of quadratic programming subproblems , obtains subproblems by using a quadratic approximation of Lagrangian and by linearizing the constraints. That is,

minimize

$$1/2\mathbf{d}^T\mathbf{H}_k\mathbf{d} + (\nabla\mathbf{f}(\mathbf{X}_k))^T\mathbf{d} \tag{C.5}$$

subject to :

$$\nabla f_i(\mathbf{X}_k)^T\mathbf{d} + f_i(\mathbf{X}_k) = 0 \tag{C.6}$$